DOCTORAL (PHD) DISSERTATION

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< The Effect of Teaching Conceptual Knowledge on Students’ Achievement in, Anxiety About, and Attitude Toward Mathematics in the Kurdistan Region of Iraq>

<2023>

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<Budapest, 2023>

Abstract

While thus far mathematics researchers have tended to concentrate on procedural knowledge, in the last few decades, there has been increasing interest in conceptual knowledge. Therefore, the present dissertation highlights the importance of teaching mathematics conceptually alongside the teaching of procedural knowledge from researchers' and educators' perspectives. In addition, it investigates how teaching for conceptual understanding affects students' achievement in, anxiety about, and attitude toward mathematics.

This study draws on interviews with thirty secondary school mathematics teachers from the Erbil city in the Kurdistan Region of Iraq, regarding their views on the conceptual aspect of mathematical knowledge. The three main aspects of the study are focused on: mathematics teacher's perspectives on teaching mathematics conceptually; mathematics teachers' need to teach conceptually, and the obstacles that face them in teaching mathematics conceptually. Furthermore, an experimental approach is utilized to evaluate 200 secondary school students from the same area. In the experimental group, conceptual teaching was the focus. While, in the control group, conventional teaching was used. Pretests and posttests for an achievement test, abbreviated Math Anxiety Scale, and Mathematics Attitude Scale were applied to both the treatment and control groups to reveal the effect of conceptual knowledge on students' achievement in, anxiety about, and attitude toward mathematics, respectively.

A thematic analysis of the interviews with secondary school mathematics teachers reveals that they believe that conceptual knowledge is as important as procedural knowledge. They believe that achieving a balance between conceptual and procedural understanding as well as connections between them, are necessary for understanding real mathematics. Furthermore, the pretest and posttest results with secondary school students show that there is a statistically significant difference in mathematics achievement between the two groups ($p < .001$). Students' attitudes toward mathematics in the treatment group developed positively. Nevertheless, teaching mathematics conceptually reduced anxiety among female students more effective than it did among male ones.

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List of Acronyms

AMAS ------------ Abbreviated Math Anxiety Scale

CVI ------------ Content Validity Index

CVR ----------- Content Validity Ratio

GDP ----------- Gross Domestic Product

MAS ------------ The Mathematics Attitude Scale

NCTM ------------ National Council of Teachers of Mathematics

OECD ------------ Organization for Economic Co-operation and Development

PISA ------------ Programme for International Student Assessment

Acknowledgments

Words cannot express my gratitude to my God, because he gives me the ability and power in this life to read, write and understand my environment.

I would like to express my deepest appreciation to my supervisors, Professor Csíkos Csaba and Professor Bernath Laszlo, for their guidance, valuable feedback, and valuable critique that they provided during my PhD. Thanks should also go to all of the staff at the Faculty of Education and Psychology at Eötvös Loránd University for their assistance throughout my PhD. study.

I would like to extend my sincere thanks to Salahaddin University-Erbil and the College of Basic Education, specifically, the administration and academic staff in the Mathematics Department for their support and cooperation.

I would like to acknowledge all the mathematics teachers and students who agreed to take part in my research and shared their experiences generously with me. Without their voluntary contributions, my studies would have not been accomplished.

I could not have undertaken this journey without the support of my family. Their patience, encouragement, and kindness are precious and meaningful during my study. In addition, my special thanks to all of my friends and colleges in Hungary and in the Kurdistan Region of Iraq, who support and motivate me during my study.

Chapter One

1. Introduction

In the introduction chapter, the rationale and background of the present study are outlined. In addition, this chapter addresses the significance of the current study, problem statement, hypothesis, purpose of the study, and research questions. The chapter is finalized with the outline of the whole dissertation.

1.1. Rational and Background

Learning mathematics has importance in our daily life. Mathematics help people to better understand their environment and it provides the power to think logically (Graeber & Weisman, 1995). We use mathematics continually in our daily life. For example, when people pay the bills, they think about the amount of reducing on their bank account, or when people want to build a new house, they need to think about the amount of materials that they need, the time, and the cost (Anjum, 2015). Furthermore, mathematics is the main component in other disciplines. According to Graeber and Weisman (1995), mathematics, as a tool, is fundamental for understanding other disciplines such as science, engineering, and economics. Butterworth, Varma, and Laurillard (2011) show that if 19.4% of lowest achieving children in mathematics in the USA were brought up to the minimum level (Level 1 in the PISA Survey), GDP would increase by 0.74%. They present the results of a large sample study (2 x 17 000 persons) showing that low math skills are grater disadvantage in a person's life than low literacy (Butterworth, Varma & Laurillard, 2011). Therefore, understanding mathematics is necessary for our daily life to make daily affairs easier.

Improving the quality of mathematics teaching and learning has become a serious subject across the globe by researchers and educators (Coburn et al., 2012; Cobb & Jackson, 2011). The important of teaching conceptual mathematics has been taken into consideration by mathematics researchers and educators (see, for example, Andrew, 2019; Baroody & Lai, 2007; Bransford et al., 2000; Clarke-Midura & Dede, 2010; Crooks & Alibali, 2014). According to Andamon and Tan (2018), the student's ability to solve mathematics problems is affected by the mathematics competence that needs basic mathematics skills such as conceptual understanding. Students with conceptual knowledge can easily transfer their knowledge into new mathematics situations.

Accordingly, to understand mathematics deeply and successfully, the learners must have conceptual knowledge (National Council of Teachers of Mathematics, 2000). Another main point for conceptual understanding is that students with conceptual knowledge can assess which procedure is more appropriate for a specific mathematical problem (Brownell, 1945; Schneider & Stern, 2010). Students will get higher scores in mathematics if they participate in thinking and exploring in the learning process rather than participating only in the learning of mathematics rules mechanically. Their achievement can be increased by providing an attractive and interesting teaching method by the teacher to engage students' interest (Kiuru et al., 2014). Therefore, teaching mathematics conceptually is central to students’ better understanding of mathematics which leads to higher achievement in mathematics (Kiuru et al., 2014). Accordingly, the mathematics education researchers’ focus has growingly shifted from procedural knowledge to conceptual knowledge (Crooks & Alibali, 2014).

1.2. The Significance of The Study

Mathematics researchers, in the last few decades, have concentrated on the necessity of conceptual teaching for students’ success in learning mathematics (see, for example, Crooks & Alibali, 2014). According to the National Council of Teachers of Mathematics (2000), for students to be successful in learning mathematics, they have to learn with understanding. The reason for students’ low performance in Calculus was the lack of conceptual understanding (Liang & Martin, 2008).

Teaching conceptually helps the learners to explain the reason behind working a particular mathematics operation in a specific way. The students do not need to depend only on remembering the rules, but they understand the mathematics situations then they apply the mathematics rules to get the solution, which helps to reduce the students’ anxiety and increase their positive attitude toward mathematics (Webb, 2017). In addition, conceptual knowledge helps students to acquire the ability to transfer their knowledge among different new mathematical situations (National Council of Teachers of Mathematics, 2000). Accordingly, the students have to be taught basic mathematics rather than just procedures (Generalao, 2012).

Educators and researchers must consider anxiety about and attitude toward mathematics because these are serious problems among students (Christiansen, 2021; Webb, 2017).

The concept of attitude is used and understood in the way Pehkonen and Pietilä (2003) defined as a psychological construct that belongs to the affective-emotional side of human personality. Anxiety also belongs to the emotional sphere of personality, but definitely to the negative side (Hembree, 1990).

Students often have mathematics anxiety, and they think mathematics is a dreaded subject (Capinding, 2022). This anxiety impedes students' development and improvement in their mathematics competence (Cribbs et al., 2021; James et al., 2013). The relation between students’ anxiety and their performance on mathematics exam is negative (Chernoff & Stone, 2012). For example, a study managed by Price (2015) investigated the characteristics of students’ anxiety and its effects on their understanding of arithmetic. The result showed the students who had low mathematics anxiety performed better and understood more the concept of arithmetic compared with students who had high anxiety. In addition, in a study conducted by Capinding (2022) on students in high school, the results found that there was a negative relationship between anxiety and mathematical performance. The students who had mathematics anxiety performed worse than those who had not. Accordingly, both mathematics anxiety and performance affect each other negatively (Carey et al., 2016; İlhan et al., 2022). Therefore, a holistic teaching approach is needed to support the education process that concentrates on both emotion and skills toward mathematics (Sorvo et al., 2019).

Students' attitudes toward mathematics have a direct impact on their mathematics achievement. Students with a positive attitude toward mathematics are more academically prepared to develop in mathematics (Capinding, 2022). In their study, Zhang et al. (2020) found that students' attitudes had a positive relationship with their mathematics performance. In a study conducted by İlhan et al. (2022), the results revealed that the relationship between students’ negative attitude and their mathematics achievement was significantly negative. Simultaneously, according to Capinding's (2022) findings, there was a negative relation between students' anxiety about and their attitude toward mathematics. Students, who were anxious about mathematics, had a negative attitude toward it. And students, who had a negative attitude toward mathematics, had more anxiety. Consequently, a positive attitude toward mathematics overcomes mathematics anxiety (Saha et al., 2020).

To sum up, students understanding of mathematics and their confidence regarding their ability can be developed by providing them interactive teaching methods in a relaxing environment that leads to improve their attitudes toward mathematics (Jennison & Beswick, 2010). Therefore, mathematics educators must focus on using a holistic teaching approach to interact the students with mathematics subjects and to improve their positive attitude toward mathematics (Turner et al., 2002).

1.3. Problem Statement

Numerous mathematics educators tend to use conventional teaching methods that concentrate on procedural teaching and neglect the important aspect of teaching which is conceptual teaching. Mathematics teachers define mathematics as provider a set of tools, such as problem-solving skills, logical reasoning, and thinking ability abstractly (Andamon & Tan, 2018). Based on their definition mathematics teachers depend on procedural teaching rather than conceptual teaching which is an insufficient approach to improving the students' mathematics competence. Students understanding mathematics conceptually, however, helps them to develop their confidence and decline their mathematics anxiety. This leads to an increase in their ability in confronting mathematics challenging tasks more easily and trustfully (Mariquit & Luna, 2017). According to a study conducted by Zaini (2005) on the teaching of conceptual knowledge, the result revealed that trainee teachers depended on algorithms, rules, and formulas to explain problems instead of evidence-based understanding. Likewise, in Saudi Arabia primary school mathematics teachers depended more on procedural knowledge than conceptual knowledge (Khashan et al., 2014). Consequently, students learn only rules and depend on them to confront in solving mathematics problems which is insufficient for solving the problems that require deep understanding (i.e., non-traditional problems). For instance, in Hussein and Csíkos (2021) study revealed that secondary school students in the Kurdistan region of Iraq had many problems with the concept of function. The study showed that students had difficulty defining the concept of function, and they tended to conflate concept image with concept definition. They could not provide a complete and clear definition of function but were only able to provide the definition partially. In addition, they had difficulty recognizing different representations of functions and conversions between different modes of representation.

According to the National Council of Teachers of Mathematics, besides students' computational ability in solving mathematics problems, basic skills should go beyond and be integrated. Because, if students only learn procedures, they face difficulties and sometimes stuck in learning higher-level mathematics. Conceptual mathematics, however, helps the learners to understand each step in problem-solving, and it opens a variety of approaches of solution for them. In addition, higher level of students' understanding of mathematics needs less practice (Brownell, 1987). Accordingly, conceptual understanding helps the students to develop a computational sense, which means more understanding of how mathematics is working. Therefore, teaching mathematics should be revised which focuses on less lecture, and more discussion with students to direct them (Curtain-Phillips, 1999).

Students' lack of mathematics proficiency does not refer to the shortage of their intelligence in the subject or their lack of ability in learning mathematics. Rather, the student's lack of mathematics competence indicates using unsuitable teaching methods by the teachers that lead the learners to lack mathematics skills, including conceptual understanding (Andamon & Tan, 2018). Using an inappropriate method of teaching by mathematics teachers made an obstacle for students. The majority of teachers define mathematics as acquiring, memorizing, and algorithms. Teachers are pleased with teaching mathematics as manipulating symbols and solving problems routinely, but not making sure that their students acquire a deep understanding or not (Hirschfeld-Cotton, 2008).

Teaching styles in schools encourage students to develop procedural understanding rather than conceptual understanding (Hussein & Csíkos, 2021). Mathematics educators want to choose the easiest way for teaching. They think if students can provide definitions and can solve mathematics problems procedurally, it means the students' understanding (Wiggins, 2014). Meanwhile, to assess mathematics competence, the students' ability to manipulate knowledge procedurally is depended on (De Zeeuw et al., 2013). For example, learners are required to find correct answers on the exam, and based on their correct answers the teacher decides who is pass or fails. Students are prepared for the national examination by teaching them and practicing questions procedurally. Therefore, the students' conceptual knowledge in mathematics does not develop because students learn in school based on the method that they are taught (Zulnaidi & Zamri, 2017). In addition, the tools used to assess them encourage teachers to concentrate only on procedural knowledge rather than conceptual knowledge (De Zeeuw et al., 2013).

Teaching mathematics procedurally, which is the common teaching method, increases concerns about students' performance and anxiety about mathematics. Khoule et al. (2017) found in their study that teaching procedurally not only does not overcome students' mathematics anxiety, but it helps to arise their mathematics anxiety. Because this method of teaching focuses on mastering rules without understanding and students should remember them in the exam. This stress of remembering the materials that students studied, in the exam, increases their anxiety (Khoule et al., 2017).

1.4. Hypothesis

The hypotheses are formed as null hypotheses. This was done for the purpose of straightforward testing them, and based on the literature the researcher's real expectations can be formed as the alternative hypotheses of the following.

- In terms of students' achievement, there will be no statistically significant difference between the control group and the experimental group.
- In terms of decreasing students' anxiety, there will be no statistically significant difference between the control group and the experimental group.
- In terms of improving students' positive attitudes toward mathematics, there will be no statistically significant difference between the control group and the experimental group.

1.5. Purpose of The Study and Research Questions

The purpose of this dissertation is to examine mathematics teachers' perspectives on the necessity of teaching mathematics conceptually and the obstacles that they face in this endeavor. In addition, it investigates how secondary school students' conceptual knowledge impacts their achievement in, anxiety about, and attitude toward mathematics in the Kurdistan region of Iraq. It aims to disseminate results and contribute to improving the teaching of mathematics.

The study is guided by the following research questions:

1. What is the importance of conceptual knowledge in teaching mathematics for students from mathematics teachers' perspectives?

   This question has four sub-questions:

   a. What is mathematics teachers' familiarity with conceptual understanding?
   b. What are mathematics teachers' perspectives on teaching mathematics conceptually?
   c. What do mathematics teachers need to teach conceptually?
   d. What are the obstacles that mathematics teachers face when teaching mathematics conceptually?
2. Does teaching mathematics conceptually affect students' achievement?
3. Does teaching mathematics conceptually affect students' anxiety?
4. Does teaching mathematics conceptually affect students' attitudes?

1.6. Outline of The Whole Dissertation

This dissertation follows a conventional structure consisting of the following chapters: introduction, literature review, methodology, result and data analysis, discussion, and conclusion. Each chapter starts with a brief introduction to provide the structure of the chapter to the reader, and it concludes with a summary that summarizes the whole chapter and provides the chapter as a coherent piece of writing.

In chapter one, the previous studies' gaps, and the importance of the current study from mathematics educators and the researchers' viewpoints are provided. This chapter explains the problem statement and research hypothesis. It also details the aim of the study and research questions. The chapter is finalized with the outline of the whole dissertation.

The second chapter focuses on a comprehensive review of relevant literature, the main terms in the dissertation are defined as conceptual knowledge, procedural knowledge, mathematics anxiety, attitude toward mathematics, and academic achievement. Throughout the literature review chapter, the importance of conceptual knowledge and its relationship with procedural knowledge are clarified in detailed. The effects of mathematics anxiety on students' achievement from previous studies' findings are provided. This chapter discusses, also, the relation between students’ attitudes toward mathematics and their performance in mathematics from educators' and researchers’ viewpoints. Then gender differences in mathematics and the meaning of academic achievement in the present study are detailed. Last but not least, to show the necessity of the present study, the contribution to the literature is provided in this chapter. The chapter is finalized by explaining the summary of the whole chapter.

The third chapter provides an elaborate description of methodology related issues including the research approach, design, and the various methods of data collection. This chapter provides the population of the study and the mechanism of selecting the sample study. It details the instruments used to collect data including interview, experiment, and questionnaire. The consideration of research ethics and evidence of validity and reliability of the present dissertation are explained in this chapter. To familiarize the reader, the education system in the Kurdistan region of Iraq is clarified. Then, the research focus and design are detailed. This chapter explains different research paradigms specifically the paradigm that the present study utilizes. The chapter is finalized by pointing out the implementation of teaching conceptually, and then the chapter summary.

The fourth chapter, named data analysis and results, contains two main sub-sections. The first sub section clarifies how data that were collected in interviews were analyzed. The second sub-section explains the data analysis of the experiment and the questionnaires. Most tables and figures that show the data analysis process of this study are presented in the result chapter. This chapter provides the mechanism of data analysis and the tools used for that purpose. Furthermore, the statistics subjects that are used to analyze the data are provided. The process of grouping the participants’ answers for the interviews is detailed. The chapter is finalized by analyzing the quantitative data, the mechanism of analyzing the data that collected from the experiment, and the questionnaires.

In the final chapter, the findings are discussed and interpreted in relation to the existing body of literature on the importance of conceptual knowledge in teaching mathematics. This chapter discusses the results of interviews separately from the results of the experiment. All the research questions are answered in this chapter accuracy in detail. In the conclusion section, the main findings of the study are focused on and remembered. This study as the previous studies has many limitations that are provided in this chapter.

The chapter is ended by providing some recommendations and suggestions for mathematics educators, researchers, and stakeholders.

Chapter Two

2. Literature Review

This chapter reviews the literature on conceptual knowledge, procedural knowledge, mathematics anxiety, and attitudes toward mathematics. It defines the main terms in the dissertation. Then the relation between conceptual knowledge and procedural knowledge, and the importance of conceptual knowledge in teaching mathematics are provided. The effect of having mathematics anxiety and negative attitudes toward mathematics on students' achievement from the previous studies' findings are detailed. Gender difference in mathematics is another aspect that is discussed and evaluated in the present chapter. The chapter concludes by remembering the key points.

One of the main parts of scientific research is the literature review which includes reviewing and analyzing existing research, literature, and theories relevant to the research topic. Robinson and Reed (2019, p. 58) define a literature review as "systematic research for published work to find out what is already known about the intended research topic". There are many advantages to literature review. Through reviewing the literature, the researcher can find out what work has been done in a specific area, and what has not been done so far. The researcher can identify the gap in his/her area which is helpful in creating a research title. By reviewing previous research and publications the researcher can avoid replicating unnecessarily existing work. A literature review assists in the clarification of terms and concepts related to the research issue, ensuring that the researcher employs proper terminology and definitions in their work. The identification of acceptable research methods and methodologies that have been employed in prior studies is helped by literature reviews. This might serve as a basis for selecting the research methodology and a justification for the chosen strategy. The findings from the research can be predicted or hypothesized by using a literature review. This can help to focus on data collecting and analysis while also guiding the research plan. Reviewing the literature enables researchers to assess the quality of prior studies and assess the strength of the body of evidence supporting a specific subject or research question.

2.1. Conceptual Knowledge

A concept is a "mental representation that embodies all the essential features of an object, a situation, or an idea. Concepts enable us to classify phenomena as belonging, or not belonging, together in certain categories" (Westwood, 2008, p. 24). Concepts are important cognitive tools that have the capacity to organize and associate with other ideas to be connected as a web of understanding, this connection leads to the formation of conceptual knowledge (Clark, 2011).

Conceptual knowledge is the comprehension of the basic thoughts and concepts that underpin mathematical processes and problem-solving. It involves the capacity to comprehend and apply mathematical notions, connections, and correlations. Understanding arithmetic conceptually entails comprehending not just how to do addition, subtraction, multiplication, and division calculations, but also the underlying concepts and connections that underlie these operations. Knowing that addition and subtraction are inverse operations, and that division is the inverse of multiplication are all necessary knowledge for this. Understanding geometry conceptually means understanding the connections between shapes and their attributes, such as the connection between a triangle's angles and a circle's attributes. Therefore, being able to think deeply about and apply mathematical concepts in many contexts allows people to have conceptual knowledge of mathematics, which is crucial for the development of higher-order thinking skills and problem-solving abilities (Frederick & Kirsch, 2011; Rittle-Johnson et al., 2015).

There are various definitions of conceptual knowledge in mathematics studies. However, these are sometimes implicitly rather than explicitly referenced. Despite these variations, mathematics education researchers tend to define conceptual knowledge as rich knowledge about relationships and connections, such as a web of knowledge (Hiebert & Lefevre, 1986; National Council of Teachers of Mathematics, 2000). In other words, conceptual understanding means understanding how all pieces of information are linked together in a network (Baroody et al., 2007; Rittle-Johnson et al., 2015). It can be defined as an "understanding of the underlying structure of mathematics the relationships and interconnections of ideas that explain and give meaning to mathematical procedures" (Faulkenberry, 2003, p. 13). In terms of its implications for teaching, conceptual knowledge means the "comprehension of mathematical concepts, operations, and relations" (Kilpatrick, 2001, p. 5). According to Reys et al. (1995, p. 21), conceptual knowledge requires "the learner to be active in thinking about relationships and making connections, along with making adjustments to accommodate the new learning with previous mental structures." Accordingly, mathematical researchers define conceptual knowledge as a relation to the connection and linkage of ideas.

In some mathematics education studies, conceptual knowledge has been defined in terms of principles. For instance, according to Baroody et al. (2007, p. 123), "conceptual knowledge is knowledge about facts (generalizations) and principles." In the classroom, "having conceptual knowledge involves the student understanding the meaning and underlying principles of mathematical concepts" (Frederick & Kirsch, 2011, p. 94). Accordingly, conceptual knowledge is sometimes called conceptual understanding or principled knowledge (Kilpatrick et al., 2001).

Furthermore, mathematics education researchers have defined this type of knowledge without reference to it being "conceptual." For example, the National Council of Teachers of Mathematics (2000, p. 118) referred to it as "an integrated and functional grasp of mathematical ideas." Robinson and Dube (2009, p. 193) explained it as "the understanding of the underlying structures of mathematics," while Lampert (2001) and Ball et al. (2001) understood it as the knowledge that stimulates the growth of mathematical algorithms. Conceptual mathematics understanding is "knowledge that involves thorough understanding of underlying and foundation concepts behind the algorithms performed in mathematics" (Andamon & Tan, 2018, p. 5). Thus, the meaning of conceptual knowledge has expanded to include the grasp of ideas, mathematical structures, and the stimulation of algorithms.

As shown above, there are various definitions of conceptual knowledge. However, the most common and preferred two definitions in the present dissertation are understanding relationships and connections between ideas, symbols, and numbers as a web of knowledge (Bolden & Newton, 2008; Dixon & Moore, 1996). And the second preferred definition is an "understanding of the underlying structure of mathematics, the relationships and interconnections of ideas that explain and give meaning to mathematical procedures" (Faulkenberry, 2003, p. 13).

2.2. Procedural Knowledge

Procedural knowledge is knowing how to do something. Procedural knowledge, which is typically related to motor abilities or muscle memory, is the understanding of how to carry out an activity or skill. It requires the capacity to carry out a series of tasks in a coordinated way in order to accomplish a particular goal. The acquisition of procedural knowledge frequently involves repetition and practice. Procedure knowledge is necessary, for instance, to ride a bicycle or play an instrument since the actions involved must be carried out in a particular order and time. There are two aspects to the definition of procedural knowledge. The first is knowledge of the formal language, which is called symbolic representation, and the second is knowledge of the rules used to complete a mathematical task (Hiebert & Carpenter, 1992). Rittle-Johnson and Schneider (2015, p. 4) defined procedure knowledge as a "series of steps, or actions, done to accomplish a goal." Furthermore, it is "knowledge of the steps required to attain various goals" (Canobi, 2009, p. 176). According to Engelbrecht et al. (2017), "a procedural approach includes algebraic, numerical calculations, employing rules, algorithms, formulae and symbols" (p. 574). Hiebert and Carpenter (1992) understood procedural knowledge as a series of actions that, if executed correctly, will lead to the right answer. Thus, all these definitions refer to the idea that knowledge relates to sequences of procedures that can be used in mathematical problem-solving. Meanwhile, the definition that I depended on in this dissertation is procedural knowledge defined as "Mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms, and definitions" (Faulkenberry, 2003, p. 13).

Several factors make procedural knowledge essential in mathematics. To start, it enables students to solve mathematical tasks quickly and precisely, especially those that are common or clearly stated. Learners can save time spent on calculations and eliminate errors by becoming familiar with the relevant techniques. Procedural knowledge is essential to developing mathematics automaticity and fluency. By repeating and rehearsing methods, students can acquire the abilities needed to execute calculations fast and accurately, freeing up mental resources for extremely hard problem-solving tasks. To think and solve mathematical problems at a higher level, procedural knowledge is required. The ability to solve problems conceptually is crucial, but it is frequently insufficient on its own. In order to use mathematical procedures and algorithms to solve complicated issues, students must also possess procedural knowledge (Engelbrecht et al., 2017; Hiebert & Carpenter,1992).

2.3. Relationship Between Conceptual and Procedural Knowledge

Mathematics educators believe that both conceptual and procedural knowledge are essential (Hiebert & Grouws, 2007; Hurrell, 2021; Rittle-Johnson et al., 2015). For instance, Kilpatrick et al. (2001) stated that procedural knowledge is primarily needed to support conceptual knowledge. Procedural knowledge is an important aspect of mathematical proficiency, it is also important to develop the ability to understand the underlying principles and relationships that drive mathematical concepts and procedures. The basis for mathematical fluency and problem-solving skill is a combination of procedural knowledge and conceptual comprehension. Therefore, conceptual knowledge and procedural knowledge are both necessary and help to strengthen each other (Hurrell, 2021), and connecting these two types of knowledge is the key to developing mathematical understanding (Aydın, 2018; Baroody et al., 2007; Hiebert & Lefevre, 1986) (see [Figure 1]). Similarly, simultaneously developing these two types of knowledge has a positive effect on mathematical competence (Rittle-Johnson et al., 2015). Accordingly, conceptual understanding is supported by algorithms and provides building blocks that can be used to clarify concepts. Conversely, students can develop algorithms through conceptual understanding (Aydın, 2014). Therefore, conceptual and procedural knowledge are often mentioned together because it is believed that they have a coherent relationship between them (Rittle-Johnson & Schneider, 2015).

However, conceptual knowledge is distinct from procedural knowledge in several respects. Conceptual knowledge has strong relationships with different parts of knowledge, whereas procedural knowledge is a conventional sequence of steps (Hiebert & Lefevre, 1986). Applying procedures mechanically that employ rules without understanding the reason might lead to getting strange and ambiguous solutions (Martin, 2009). Moreover, procedural understanding only concerns algorithms and facts, while conceptual knowledge confirms students' ability to link mathematics across disciplines and critical thinking with the communication of critical components of mathematics (Hiebert & Lefevre, 1986; Linn, 1994).

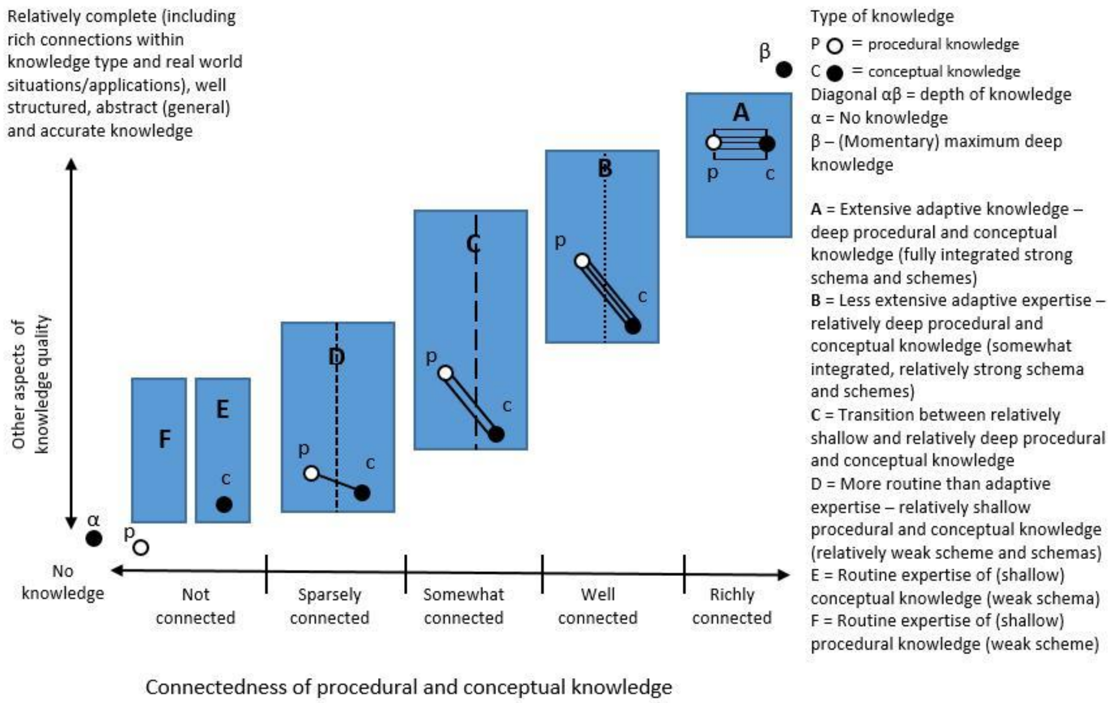

Figure 1:The mutually dependent relationship between procedural and conceptual knowledge suggested by a model of adaptive reasoning - Baroody et al. (2007, p. 124)

Conceptual and procedural knowledge are not independent (orthogonal), even the application of simple algorithms to new situations requires the integration of procedural and conceptual knowledge. On the other hand, the depth of understanding depends on the extent to which procedural and conceptual knowledge are linked. In Figure 1, the depth of knowledge ranges from no knowledge (the α point) to maximum knowledge (the β point) along the diagonal (Baroody et al., 2007).

Mathematics education studies support the perspective that understanding mathematics requires students to make connections between procedures, facts, concepts, relationships, and mathematical ideas (Hiebert & Carpenter, 1992; Moschkovich et al., 1993; Skemp, 1976, 1989). According to Siregar and Siagian (2019), mathematics makes sense for students if they know the connections between the concepts. The ability to connect mathematical conception meaningfully is essential for students at every level of education (Siregar & Siagian, 2019). This ability helps them to use the mathematical concepts that have been learned as basic knowledge to understand new concepts (National Council of Teachers of Mathematics, 2000). In addition, mathematical connections can be counted as a consequence of constructivist theory in mathematics learning and is a building of a mental network organized such as a spider's web that nodes can represent the pieces of information, and threads as the connections between them (Hiebert & Carpenter, 1992). Then for learning and thinking about connections between mathematics concepts, it is very important to look at mathematics as a whole (Siregar & Siagian, 2019). Consequently, understanding connections is fundamental in teaching mathematics (Eli et al., 2011). In this respect, if mathematics teachers focus on mathematical connections, then students acquire an interconnected understanding of mathematics (Evitts, 2005).

2.3.1. Different Terms for Conceptual and Procedural Knowledge

Many different terms had been used for different types of knowledge (see Table 1). For example, the terms instrumental, syntax, episodic memory, teleologic, and mechanical understanding are used for procedural knowledge. And the terms relational, principles, semantic, declarative, schematic, and meaningful understanding are used for conceptual knowledge.

There is a similarity between conceptual knowledge and procedural knowledge with the concepts of relational understanding and instrumental understanding, respectively (Skemp, 1976). Relational understanding is described as the ability to figure out a specific rule, while instrumental understanding is described as the ability to apply a rule to solve a mathematics problem without understanding how it works (Jones, 2011). Instrumental mathematics consists of the learning of a steady plan in problem-solving, by which students could find their way from specific starting points to finishing points. Instrumental understanding is the student’s ability to apply the mathematics rules to solve the problems regardless of knowing the reason for working strictly according to the rules (Skemp,1976). In contrast, relational understanding in mathematics contains the construction of a conceptual structure in which an unlimited number of plans for problem-solving can be produced by its owner (Skemp, 1976). Relational understanding helps the students to use an appropriate procedure to solve mathematics problems and logical reasoning (Patkin & Plaksin, 2018; Utomo, 2020). It also provides the students capability to deduce appropriate procedures in mathematics problem-solving (Minarni et al., 2016; Skemp, 2006). This theoretical idea discovers the processes for obtaining knowledge and provides instruments to solve problems for the learning aspect (Skemp, 1976).

Table 1:Differing Terms for Procedural and Conceptual Knowledge - Hiebert & Lefevre (1986, p. 14)

		Procedural	Conceptual
Skemp	1976	Instrumental	Relational
Piaget	1978	action	Conceptual understanding
Gelman & Gallistel	1978	Skills	Principles
Resnick	1982	Syntax	Semantic
Tulving	1983	Episodic memory	Semantic memory
Anderson	1983	Procedural	Declarative
Van Lehn	1983	Teleologic	Schematic
Baroody	1984	Mechanical	Meaningful

2.3.2. Instrumental Mathematics

Mathematics teachers use instrumental mathematics rather than relational mathematics because of some reasons. One of the points is, instrumental mathematics is much easier and quicker than relational mathematics in some topics, such as multiplying two minuses equal to plus or dividing two fraction numbers. Another point, in instrumental mathematics less knowledge is involved than in relational. Therefore, the students feel more comfortable with instrumental mathematics (Skemp, 1976).

However, there are many advantages to relational mathematics. Relational understanding helps the learners to adapt their knowledge into new mathematics tasks, this helps the students to generate an original idea which is the achievement of the learning aim (Skemp, 2006). In addition, relational understanding helps students to remember the rules easily. For example, when students learn the area of a triangle, they learn the rules of triangle and rectangle and find the relation to the area of a triangle. These connections help the learner to remember the rules as a part of the whole. In that case, students not only understand relationally but also, they will be active in finding new areas such as the root of the tree that extends in all directions (Skemp, 1976). Consequently, relational understanding should be encouraged, and conceptual structures that contain relevant concepts should be developed to achieve this understanding (Star & Stylianides, 2013).

In summary, students must be given opportunities to connect these two types of knowledge (Ministry of Education, 2001). Because they must have a variety of perspectives on mathematics in problem-solving and build connections between them to have better performance (National Council of Teachers of Mathematics, 1989).

2.4.Importance of Conceptual Knowledge in Teaching Mathematics

The students struggle with mathematics, not because of their inability, but it refers to the teaching method that mathematics teachers follow. Conceptual understanding skills are required for students to get the competence to solve a variety of mathematics problems successfully. Therefore, teaching mathematics conceptually is necessary for students to absorb mathematical subjects successfully (Andamon & Tan, 2018). While the lack of conceptual knowledge leads to a variety of challenges for students (Tekin-Sitrava, 2017).

Investigating conceptual knowledge helps learners gain procedural knowledge. In Lauritzen's study (2012), students who scored highly on conceptual tasks also scored highly on procedural tasks. However, a low level of conceptual knowledge was recorded among first-year students in the mathematics department at the College of Education (Saeed, 2016). When students are asked to solve a mathematical problem, they can use processes to find the correct solution despite lacking an understanding of "how" and "why" (Barr et al., 2003). Therefore, "the results support the genetic view that procedural knowledge is a necessary but not sufficient condition for conceptual knowledge" (Lauritzen, 2012, p. 13)

Many studies have indicated that a lack of conceptual knowledge leads to a variety of challenges. For example, students have difficulty with algebraic concepts such as algebraic expressions due to a lack of conceptual knowledge (Tekin-Sitrava, 2017). In Rittle-Johnson and Alibali's (1999) study, equivalent tasks were provided to students, and they were asked to decide which one was correct and which one had no meaning. The study found that 86% of participants failed to solve the problems because they lacked basic arithmetic skills. In addition, a study by Carlson (1998) found that university students were unable to solve an unconventional problem in the development of the concept of a function. Specific problems have been identified in the research. For example, in calculus, derivation was found to be particularly difficult for most undergraduate students to understand (Saha et al., 2010). According to Willingham (2009), students did not understand the base of ten numbers system totally, and only twenty-five percent of sixth-grade students have a deep understanding of the equal sign. These difficulties are believed to result from students’ lack of conceptual understanding of the concepts (Saha et al., 2010; Willcox & Bounova, 2004). Therefore, a lack of conceptual knowledge is a reason for students’ weak performance in mathematics (Knuth et al., 2005).

Students have conceptual knowledge if they can provide logical relationships between concepts (Mariquit & Luna, 2017). For example, after students' number sense has been developed, they can effectively solve mathematical tasks. Task 18*3 might be changed to double 9*3. The ability to make this change is called the flexibility of thought (Mariquit & Luna, 2017).

To develop conceptual knowledge, learners are trained to solve mathematical problems that contain new ideas. This technique helps them to think deeply and to apply previously learned information to solve new mathematics tasks. Training students to solve challenging tasks helps to improve their critical thinking skills, thereby leading to better performance in mathematical problem-solving (Mariquit & Luna, 2017). Students’ conceptual understanding develops if they are challenged to think and give a reason, then communicate their ideas orally or in writing with others (Hirschfeld-Cotton, 2008). A study managed by Khoule et al. (2017) revealed how teaching mathematics conceptually affects students’ performance in mathematics exams. An experimental approach was used. Two groups were formed. One was taught conceptually, while the other was taught procedurally. Two quizzes were developed: a conceptual quiz and a procedural quiz. The results showed that the conceptual group performed better than the procedural group in both the conceptual and procedural quizzes. Nonetheless, the procedural group practiced procedural mathematics problems more frequently than the other group. The test questions were designed to reveal the participants’ knowledge about the subject matter. The results showed the procedural group had less understanding of the subject matter than the conceptual group. This suggests that the conceptual group had a greater ability to reason logically, formulate solutions, and understand mathematics flexibly. Compared to the procedural group, they better used their knowledge as a tool in problem-solving. Therefore, there is a positive correlation between the conceptual understanding of mathematics and academic achievement (Zakaria et al., 2010).

2.5. Metacognition and Conceptual Knowledge

Results from research on metacognition may provide a powerful and sound basis for enhancing students' knowledge at the same time as enriching their conceptual understanding and reducing their anxiety. Conceptual knowledge of mathematics assumes that students understand why, when, and how to use mathematical procedures. This kind of knowledge belongs to what the literature usually labels as metaknowledge or metacognition. To simplify the distinction between metacognition and cognition, metacognition is the process of monitoring and controlling students' needs and thinking about the problem-solving process. Cognition, in contrast, comprises more or less automatized processes. Metacognition is thinking about the thinking process; it is also defined as awareness and management of the cognitive process (Kuhn, 2000).

Among educators and researchers, metacognition-based educational approaches have become effective for teaching and learning mathematics. This is because these approaches have positive effects on the construction of new knowledge and because they develop students' mathematical skills (Du Toit & Kotze, 2009). The metacognitive components of mathematical thinking may fulfill several different roles in the process of human development; the case of arithmetic performance has been analyzed by Csíkos (2022). To summarize, both metacognitive and nonmetacognitive components have important functions in the context of human development. These functions vary according to the differences between individuals, task demands, and the context of mathematical problems.

Consequently, it is very important for mathematics teachers to balance between conceptual and procedural approaches, both of which are necessary to improve students' performances (Hill et al., 2008). Nevertheless, according to the National Council of Teachers of Mathematics (2000), mathematics teachers must explain mathematics subjects conceptually before they explain them procedurally. It is believed that educators who wish to provide more meaningful mathematics knowledge to learners should start with visual models and end with a symbolic model (Ketterlin-Geller, 2007; National Council of Teachers of Mathematics, 1989). Because students who were instructed mathematics conceptually first and then procedurally outperformed students who taught the other way around (Pesek & Kirshner, 2000).

2.6. Students’ Problem with Conceptual Knowledge in Kurdistan Region of Iraq.

A study was conducted by Hussein and Csíkos (2021) to investigate secondary school students’ understanding of the concept of function in the Kurdistan Region of Iraq. To carry out the investigation, a questionnaire was administered in secondary schools in Erbil city.

The test was designed to investigate three main aspects of students' understanding: students' knowledge of the definition of the function; students’ ability to recognize different representations of function; and students' ability to convert a function from one type of representation to another.

2.6.1. The Three Main Aspects of Students' Understanding

- **Students' Understanding of the Definition of Function**

The study has revealed students’ difficulties in providing a proper definition for the concept of function in that the majority provided an ambiguous definition. This suggests that although students may have some idea about the definition of the function, it is incomplete. As a result, students defined the concept of function based on the concept image instead of the concept definition, or they only provided a partial definition.

The students’ responses to the second task (see appendix H) demonstrate a better understanding of the definition of the function than in the first task. It seems that respondents were able to represent the definition of the function by a correspondence of sets. The students’ answers indicate that they concentrated on the part of the definition of function and they forgot to think about another part of it, in which all elements in the domain have to have a correspondence with elements in the co-domain.

Results from the first three tasks in the questionnaire revealed difficulties with giving a proper interpretation of the function and evidence that this definition was not fully understood. This result is consistent with the findings of Elia et al. (2007) where the majority of students provided ambiguous interpretations, and just 35% were able to provide an accurate description. Tall and Vinner (1981) found that students depended on a concept image to define the function and as a result, they selected functions that did not have a gap in their graph as a continuous function.

There is a strong relationship between students' ability to provide a proper definition of function and their skill in solving the questions that relate to this definition. For instance, Elia et al. (2007) found that 91.1% of students who provided the right definition of the function successfully solved the other tasks that related to the definition. Therefore, understanding the definition of function is essential for students to have the ability to solve tasks related to it.

- **Students' Ability to Recognize Different Representations of Function**

Tasks 4, 5, and 6 in the questionnaire were designed to test students' ability to recognize different representations of the function (see appendix H). The findings revealed three main difficulties for students in this area. Firstly, students had difficulties with the symbol sense and interpretation of the symbol f(x). Secondly, students had a problem with finding a relationship between the definition of function and the graph representation. Finally, students were not entirely familiar with ordered pair representations.

These findings are consistent with the conclusions of previous studies. A study conducted by Vinner and Dreyfus (1989) revealed that students had a problem with terminology that relates to an understanding of the concept of function. Students do not comprehend that f(x) is only a symbol that can be easily replaced by any other symbol. According to Dreyfus and Eisenberg (1982), students have difficulties with representing the symbol f(x) and misconceptions about f(x) as a notion. Kurt and Cakiroglu (2009), and Schoenfeld et al. (1993) indicated that students had difficulties coordinating between the different representations of functions such as graphs, tables, and equations. Markovits et. al. (1986) found that students faced difficulties and confusion when presented with functions that are represented by a set of points (ordered pairs).

Another problem with graph representation that arose is that respondents could not find a relation between the definition of the function and the graph representation. As in previous studies (see, for example, Kurt & Cakiroglu, 2009) this task reveals that the students faced difficulty with the graphical representation of the function. Kurt and Cakiroglu (2009) also found that the majority of participants did not have enough ability to understand and recognize different representations of the function. The researchers pointed out that different representations are not considered necessary in the school curriculum and argued that this is the main reason why students are unable to recognize different representations of the function, specifically, graph representation.

On the other hand, participants in the study seem to be more comfortable with graphical representation. This result is consistent with findings in Elia et al.'s (2007) study in which the majority of participants recognized the graph of the function. In other words, participants performed better in graphical representation compared with other representations. This finding is in stark contrast with Kurt and Cakiroglu (2009) who found that graphical representation was the most challenging for the students.

- **Students' Ability to Convert Function Between Diverse Modes of Representation**

The last three tasks in the questionnaire focused on students' ability to transform a function from one type of representation to another. According to the findings, participants had difficulties in conversion between different representations of the function, the most challenging being from graphic representation to algebraic representation.

This finding is in line with Sfard's (1992) study in which students were unable to make a bridge between algebraic representation and graphical representation of the function. Kerslake (1986) indicated that mathematics teachers and university students in the mathematics department faced problems with conversion between different representations of the function. According to Gagatsis and Shiakalli's (2004) study, students found some transitions easier than others. The transition to the graph was more comfortable than the transition from the graph, and the most difficult conversion was found to be the transition from graphical representation to verbal and algebraic representation. In addition to this, Kaldrimidou and Ikonomou (1992) found that Greek students sidestepped the use and interpretation of the graphical representation, preferring instead to use algebraic representation. In contrast, in Hitt’s (1998) study, which was conducted with mathematics teachers, the highest rate of correct responses was in the transition function from the graphic representation to other representations.

2.6.2. Relation Between the Three Perspectives

In the study, the majority of the participants who provided the correct definition of function and understood it, also performed well in recognizing different representations of function and conversion between them. About 78.5% of those who provided a correct definition of function and understood it, were also able to provide the right answer when it came to recognizing different representation tasks in the questionnaire. Also, 71.4% of participants who provided a correct definition of function, were able to respond correctly when it came to converting function between diverse representation tasks in the questionnaire.

The study revealed that students have difficulties in providing a proper definition of function and understanding it. Three main problems were identified in students’ responses to the questionnaire. Firstly, students confuse the concept image with the concept definition of the function in that they defined a function based on a concept image rather than a concept definition. Secondly, most students provided a partial definition of function and neglected the other part, resulting in ambiguous definitions. Finally, many students who answered the tasks correctly, demonstrating the students’ understanding of the definition of the function, could not justify their answers adequately. This suggests that students have misconceptions with regards to the definition of the function.

As the results of this study show, students have difficulty in recognizing different representations of function. Two main reasons have been identified for this difficulty. Firstly, students had a problem with symbol sense. Secondly, students had difficulties in finding a relationship between the definition of function and different representations of it. However, it seems that the students are more comfortable with graphical representation because, in this study, the students' performance in the graphical representation task was better than in other tasks. As a consequence, it can be asserted that the students did not grasp a wide range of aspects when identifying different representations of the function.

Furthermore, students had difficulties in the conversion function between different modes of representation, the most difficult being the conversion function from graphic representation to algebraic representation. The findings in this study have proved that the students' ability to provide a correct definition of the function has a positive relationship with their ability to recognize different representations of function and conversion between them.

Based on the findings, secondary school students in Kurdistan have difficulty in providing a proper definition of the function, recognizing diverse modes of representation of functions, and converting between different representations of the function. This refers to their understanding of mathematics procedurally rather than conceptual understanding (Hussein & Csíkos, 2021).

2.7.Mathematics Anxiety

Mathematics anxiety is a negative feeling of distress that arises when confronting mathematics problems (Ashcraft, 2002; Jansen et al., 2013). It has been defined as "feelings of tension and anxiety that interfere with the manipulation of numbers and solving mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). "A feeling of tension, apprehension, or fear that interferes with math performance" is called mathematics anxiety (Ashcraft, 2002, p. 181).

Mathematics anxiety has been a central issue in education for many decades (Khoule, et al., 2017). Dreger and Aiken (1957) first suggested that there is a specific type of anxiety called numerical anxiety. Fifty years ago, Richardson and Suinn (1972) produce the first instrument to measure mathematical anxiety. Their questionnaire was developed over the following decades by shortening the original 98-item questionnaire (see, for example, Hopko et. al, 2003). One-third of those who consult a university student counselor have mathematics anxiety (Richardson & Suinn, 1972). More than 30% of 15 year-old students in OECD countries reported mathematics anxiety in the 2012 PISA survey (OECD, 2013). A study conducted by Curtain-Phillips (1999) on mathematics anxiety found that mathematics anxiety was common among students. According to Rossnan (2006), children struggle with mathematics anxiety that hinders their ability to understand mathematics as a part of their daily lives. Also, a study conducted by Jackson and Leffingwell (1999) on first-year college students, revealed that a huge amount of the participants had mathematics anxiety. Furthermore, it has the main role in students choosing their future careers. Majors that include high mathematics requirements are avoided by university students who have mathematics anxiety (Lefevre et al., 1992; Widmer & Chavez, 1982). Many people's fears of mathematics lead to hinder them from pursuing specific professional opportunities (Tobias, 1993). Therefore, it is vital to reduce students' mathematics anxiety because it is a barrier to improving their mathematics performance (Capinding, 2022).

2.7.1. Role of Teachers

Students' mathematics anxiety generally develops in response to previous bad experiences. Mathematics teachers with anxiety could transfer it to their students (Lau et al., 2022; Vinson, 2001). Nevertheless, the majority of researchers believe that mathematics anxiety is created in the classroom (Finlayson, 2014; Lerner & Friesema, 2013). Students' mathematics anxiety originates with low levels at the very beginning of school. After students are unable to do certain mathematics problems, their anxiety increases step by step (Shores & Shannon, 2007). Gaps in students' mathematics development appear. These gaps significantly increase mathematics anxiety, which will remain a permanent block until the learners confronted it (Shores & Shannon, 2007). Another reason behind students' mathematics anxiety is, they receive negative feedback many times from their teacher, which leads to avoiding mathematics courses and mathematics activities (Jameson, 2014). As a result, their mathematics anxiety is developed since they get scariness about not passing mathematics courses.

2.7.2. Role of Parents

Parents can pass math anxiety to their children directly or indirectly. Parents pass mathematics anxiety to their children directly by talking about the difficulty of mathematics and warning their children to not pass mathematics class if they do not make hard work. While indirectly, some parents were suffering in mathematics classes during their school. Just by talking about their bad experience with mathematics, they grow anxiety for their children (Lau et al., 2022; Wang et al., 2014). Therefore, anxiety problem in mathematics education leads researchers to conduct more studies on mathematics anxiety, students' attitudes toward mathematics, and students' understanding of mathematics (Andamon & Tan, 2018).

2.7.3. The Relation Between Mathematics Anxiety and Mathematics Achievement

The strong negative relationship between mathematics anxiety and mathematics achievement has been recorded in many studies (see, for example, Ashcraft, 2002; Ashcraft & Moore, 2009; Hembree, 1990; Ma, 1999). Mathematics anxiety poses an obstacle to students' performance in mathematics courses (Ashcraft & Moore, 2009; Vinson, 2001). For example, students can forget information and lack self-confidence as a result of their mathematics anxiety (Tobias, 1993). A study by Ashcraft and Kirk (2001) revealed that it was difficult for students with mathematics anxiety to concentrate their attention on the tasks at hand. Their distracted thoughts prevented them from developing their mathematics competence. According to Meece et al. (1990), students with mathematics anxiety in grades 7–9 had poor self-confidence toward mathematics, and they failed to enroll in advanced mathematics education courses. Gunderson et al. (2018) revealed in their study that anxiety about mathematics had a negative impact on mathematics achievement, and low mathematics achievement indicated high mathematics anxiety for students. Jamieson et al. (2021) stated that students' exam performance was affected negatively by high mathematics anxiety. In İlhan et al.'s (2022) research on lower secondary students indicated that those who had higher mathematics anxiety were less successful in performance than those who had lower anxiety. Therefore, high mathematics anxiety in students can be predicted to decline their ability in their mathematics competence (Ashcraft & Moore, 2009; Jansen et al., 2013). And there is a negative correlation (r= - 0.25 to r= - 0.40 depending on the task) between the student's performance in mathematics and their anxiety (Hembree, 1990). However, mathematical anxiety does not always reduce performance. For high intrinsic motivation, higher math anxiety improves math performance while for low intrinsic motivation performance actually worsens as anxiety increases (Wang et al. 2015).

2.7.4. The Relation Between Teaching Conceptually and Mathematics Anxiety

Teaching mathematics conceptually is important to decrease students' mathematics anxiety. A study conducted by Khoule et al. (2017) revealed the relationship between methods of teaching mathematics and students' mathematics anxiety. The anxiety of students who had been taught conceptually was compared to that of those who had been taught procedurally. The results showed a statistically significant relationship between students' anxiety and teaching for conceptual understanding. Teaching for conceptual understanding reduced students’ mathematics anxiety to a greater degree than procedural teaching. In addition, mathematics anxiety is affected by students' mathematics ability and their weakness in mathematics competence (Tsui & Mazzocco, 2006). When students do not understand mathematics subjects clearly, their lack of competence will increase which leads to creating mathematics anxiety. Therefore, conceptual teaching besides procedural teaching is necessary to increase the students’ competence in mathematics to avoid growing their anxiety. Otherwise, teaching mathematics procedurally alone not only failed to reduce mathematics anxiety, but it increased anxiety (Skemp, 1971; Khoule, et al., 2017).

2.7.5. Gender Difference and Mathematics Anxiety

Gender differences in mathematics performance have long been in the focus of international comparative studies (see, for example, Hyde & Mertz, 2009). The issue is studied primarily with the intention to reveal a possible factor of inequity in an educational system independently of the form of education (coeducation vs segregated education).

Different results have been found in studies about gender differences among students with mathematics anxiety. Price (2015) found that female students experience higher levels of mathematics anxiety than male ones. Female students experience less enjoyment of mathematics and less confidence in mathematics class; they also report their mathematics anxiety more often. It can be said that female students have more mathematics anxiety than male ones (Beesdo et al., 2009; Geist, 2010; Hembree, 1990). Nonetheless, some studies have revealed that both men and women experience the same level of mathematics anxiety (Jameson, 2014; Ma, 1999; Wood, 1988). A study conducted by Jameson (2014) on mathematics anxiety among students from kindergarten to grade 6 found no differences between genders. In the region where the current study took place, one preliminary study has been conducted on the topic by Hasan (2021) finding no significant difference in math performance in a university sample. Different results have been found in studies about gender differences among students with mathematics anxiety, namely, girls tend to possess a higher level of anxiety resulting in poorer math achievement. From this respect, it is very important to acknowledge and study the gender issue in mathematics education.

2.7.6. Starting and Developing Students' Mathematics Anxiety

For the early identification of mathematics anxiety, it is necessary to specify the stage of commencement for students' mathematics anxiety and assign the characteristics of mathematics anxiety (Ramirez et al., 2013). According to some studies, mathematics anxiety could be observed in sixth-grade children, when they face complexity in their mathematics curriculum (see for example, Ashcraft et al., 2007). While Harari et al. (2013) revealed that mathematics anxiety starts by students in the first grade. However, the majority of the researchers point to third grade as a starting stage of mathematics anxiety by students (Wu et al., 2012). I believe that it is possible for students' mathematics anxiety to start at any grade of school if they misunderstand mathematics subjects. Accordingly, mathematics anxiety can happen at any level of education primary school, high school, or college level (Barroso et al. 2021; Tobias, 1993).

The most popular characteristics of students' mathematics anxiety are fear, lack of confidence, and panic (Buxton, 1981; Gresham, 2007). According to Tobias (1993), emotional response to solving a mathematics problem, and discussing a mathematics subject can be included in mathematics anxiety.

2.8. Attitude Toward Mathematics

Research on the role of students' attitudes toward mathematics, along with challenges to that role, has attracted the attention of mathematics educators and educational researchers (Chen et al., 2018). Attitude is a psychological propensity that is expressed by evaluating a specific entity with approval or disapproval (Fisher & Rickard, 2008). In addition, "attitude is a relatively stable psychological tendency toward a particular idea, object, or entity with a certain degree of positivity or negativity" (Clore & Schnall, 2005, p. 2, cited in Sunghwan & Taekwon 2021). According to Okpala (2005), attitude is a belief that connects with events and objects.

Students' different experiences with mathematics form their attitudes toward it. They develop a positive or negative attitude because their accumulated experiences with a subject affect their psychological state (Sunghwan & Taekwon, 2021). A positive attitude enhances students active performance which leads to success, while a negative attitude has a reflection of nonparticipation as an activity that leads to failure (Abim, 2009). A study conducted by Uwase and Edoho (2018) revealed that primary school students with positive attitudes toward learning mathematics performed well in mathematical tasks. Students with positive attitudes enjoy studying and practicing mathematics, which increases their competence (Aiken, 1970; Andamon & Tan, 2018; Ashcraft & Kirk, 2001). According to Mullis et al. (2020), students with a positive attitude toward mathematics were interested in participating in mathematics courses and spent more time studying mathematics than students with a negative attitude. Furthermore, the students with positive attitudes provided better performance in specific tasks in mathematics (Stephen & Evans, 2000; Uwase & Edoho, 2018). In contrast, students with a negative attitude toward mathematics perceived mathematics as an unnecessary subject and felt afraid to participate in courses dedicated to it (Guo et al., 2015; Wigfield et al., 2016). A study conducted by Jennison and Beswick (2010) on attitudes toward mathematics revealed that students with negative feelings about mathematics performed poorly, and they have inappropriate feelings about the subject. Consequently, students with positive attitudes toward mathematics perform much better than those with negative attitudes (Papanastasiou, 2000). Therefore, to succeed in mathematics, it is important to maintain a positive attitude toward it (Dowker et al., 2012). From this perspective, the relation between students' performance in mathematics and their attitude toward it is positive significantly (Andamon & Tan, 2018).

Mathematics teachers should consider students' cognitive and emotional needs (In McLeod, 1992). Students' mathematics anxiety inhibits their cognitive development by creating negative attitudes that affect their long-term futures (Wu et al., 2012). Therefore, if mathematics instruction shifts from only performing algorithms to critical thinking and conceptual understanding, then students' attitudes toward mathematics will improve (Hirschfeld-Cotton, 2008).

2.9. Gender Difference in Mathematics

Different results were found by the mathematics researchers on the gender difference in mathematics performances. Hall et al., (1999) found that in the final year of secondary school, the gender difference in mathematics achievement was found. However, Mullis et al., (1997), revealed that students' grades in primary and middle school between males and females were very close.

Some differences between genders arise at some points in students' performance in mathematics. Females performed better on mathematics tests that consist of computation, while males performed better in problem-solving, and there is no significant difference between them in understanding mathematics concepts (Hyde et al., 1990). Furthermore, males feel more confident than females in learning mathematics (Fennema & Sherman, 1978). Boys perceive mathematics to be more necessary than girls do, and boys spend more time on mathematics homework than girls (Samuelsson & Samuelsson, 2016). This makes boys generally score higher than girls on the standardized test of mathematics (Cleary, 1992; Gallagher & Kaufman, 2005). In Engelhard's (1990) study on 13 years old students, found that boys had a better performance than girls in changing the level of complexity in mathematics content. According to Campbell and Beaudry's (1998) study conducted on mathematics achievement in grade 11th, the result showed that males earned higher scores than females, while this difference between genders was less appeared in grade 10th.

However, some studies show that females performed mathematics better than males. For example, In Kimball's (1989) study, females could earn more grades than males in mathematics test. Furthermore, Alkhateeb (2001) revealed in a study conducted in the United Arab Emirates on high school students, females earned higher grades in mathematics achievement test than males. The females got more grades than males in terms of enjoying learning mathematics and participating in mathematics courses. It seems that girls have a better view of the importance of mathematics in their future success (Azina & Awang, 2009).

I believe that there are no significant differences between boys and girls in mathematics performance (Farooq & Shah, 2008). There is no difference in the neural functioning system between males and females during mathematics development (Kersey et al., 2019). However, some factors have affected to arise the differences, for example, self-confidence, spending more time on learning mathematics, belief in its necessity in future career life, and it might be socio-cultural has affected the society thinking that mathematics is more for boys domain than girls (Johnson, 1974), or, males do better than females in mathematics. This view possibly has affected students' attitudes toward and performance in mathematics. This is supported by the findings of Beilock et al (2010), who found that in first graders, although there was no evidence of mathematics anxiety in either boys or girls at the beginning of the school year, by the end of the school year math anxiety had developed in girls who believed the stereotype that boys were good at mathematics and girls at reading.

2.10. Academic Achievement

Academic achievement is a “specified level of attainment or proficiency in academic work as evaluated by the teachers, by standardized tests or by a combination of both” (Bhat & Bhardwaj, 2014). Better academic achievement means that students tend to excel (Robiah, 1994). In the present study, students’ achievement refers to their success in an exam, and the exam questions are based on an eighth-grade curriculum (Zulnaidi & Zamri, 2017). In this study, the terms achievement and performance have been used interchangeably. Distinct definitions may be found in the literature, such as achievement often refers to a kind of summative assessment while performance is a more neutral term in this respect.

2.11. Contribution to The Literature and The Novelties

The effects of procedural teaching on students’ performance in and anxiety about mathematics have been revealed by many previous studies (Ramirez et al., 2013). There are, however, fewer studies on the relationship between students’ performance in mathematics and having conceptual knowledge (Price, 2015). For example, according to Zakaria et al. (2010), there is a positive association between conceptual knowledge and academic achievement in mathematics. In addition, some studies found the effect of conceptual teaching on students’ anxiety in mathematics. For instance, according to Khoule et al. (2017) teaching conceptual mathematics decreases students' mathematics anxiety which leads to an increase in their performance. However, a study such as the present study that concentrates on how conceptual knowledge affects the three variables, students’ achievement in, anxiety about, and attitude toward mathematics, could not be found in the scientific literature.

In the region where the current study took place, one preliminary study has been conducted on the topic by Hussein and Csíkos (2021) revealed that secondary school students in Erbil city had many problems with the concept of function. The study showed that students had difficulty defining the concept of function, and they could not recognize different representations of functions. However, a study that concentrates on how conceptual knowledge affects the students' anxiety or students attitudes toward mathematics could not be found in the area. Therefore, the present study will try to investigate, on one hand, how conceptual teaching impacts students' achievement in, anxiety about, and attitude toward mathematics in secondary school students in Erbil-Iraq. On the other hand, it tries to investigate the importance of conceptual knowledge in mathematics teaching from the perspectives of mathematics teachers in the same area.

2.12. Chapter Summary

This chapter aimed to familiarize the studies that have been done so far on the relevant subject to the present study. It also discussed the most recent studies' finding about teaching mathematics conceptually and its effects on students' performance in, anxiety about, and attitude toward mathematics.

The definitions of the main terms in the present dissertation were provided, such as conceptual knowledge, procedural knowledge, mathematics anxiety, students' attitude toward mathematics, the gender difference in mathematics, and academic achievement. Furthermore, the relation between conceptual knowledge and procedural knowledge was explained. Then, in the chapter, the importance of conceptual knowledge in teaching mathematics from previous literature was discussed. Contribution to the literature and the novelty of this study is another main point that was detailed in the chapter. The chapter concluded by mentioning the main points.

Chapter Three

3. Methodology

Methodology is one of the essential parts of academic study that "shows how research questions are articulated with questions asked in the field" (Clough & Nutbrown, 2007, p. 32). This chapter clarifies the study population and the sample study. The instruments used for collecting data are detailed, mainly, interview, experiment, and questionnaire. Research ethics, which is one of the main aspects of scientific research that should be taken into consideration, is explained in the present chapter. Then, the education system in Kurdistan is clarified. Next, the research focus and design are talked about. Finally, the implementation of teaching conceptually, and the research paradigm are explained in the present chapter.

3.1. Methods and Methodology

There is a difference between research methods and research methodology. Research methods are the tools used in conducting research (Cohen et al., 2007). While "research methodology provides the reasons for using a particular research recipe" (Clough & Nutbrown, 2007, p. 23). The methodology is the process of solving research problems in a systematic way and showing the scientific approach of the study (Kothari, 2004).

3.2. Study Population

The first step in sampling is to visibly define the target population (Taherdoost, 2016). A study population is defined as "the set or group of all the units on which the findings of the research are to be applied" (Shukla, 2020, p. 3). The population for this study is secondary school mathematics teachers in Erbil city in the Kurdistan region of Iraq. In addition, the population study is students in grade 8^{th} in the same area. They are 14 years old. There are two main reasons behind choosing this area. Firstly, based on the researcher's experience, learners have suffered with understanding mathematics.

Secondly, the researcher is originally from Erbil, which made it easier to contact teachers and students, and persuade them to participate in this study as the sample study.

3.3. Participants

The sample study should be representative of the population of the study (Taherdoost, 2016). Individual semi-structured interviews were managed with secondary school mathematics teachers in the Kurdish Region of Iraq to investigate the research questions (Bryman, 2004). An online blog was made by the researcher that contained all details about the present study, namely, the title, research aim, the significance of the research, definitions, and call for voluntary participation. Subsequently, the blog link was sent to mathematics educators in Erbil, and they were contacted through phone, email, and social media. Consequently, the researcher recruited 30 secondary school mathematics teachers for the sample study. The researcher chooses the participants based on a set of basic criteria; they had to cover a range of geographical locations in the city and have different years of teaching experience. Most participants were male; in total, there were 11 women and 19 men. They have a bachelor certificate in mathematics. All of them had good enough teaching experience in mathematics, four participants had up to five years of teaching experience and the rest had over six years of teaching experience. See [Table 2].

Table 2:Participants' Experience with Mathematics Teaching

Years of experience	1–5	6–10	11–15	16 and over
Number of participants	4	6	13	7

Furthermore, two hundred students in grade 8 participated in this study, 110 female and 90 male. They were 14 years old. The researcher used Purposive sampling to select three public secondary schools in Erbil, in the Kurdistan region of Iraq. According to the previous studies, on average, taking 200 students as a sample study is sufficient for this kind of study (see, for example, Andamon & Tan, 2018; Krejcie & Morgan,1970). In the schools in Erbil, each class contains approximately 30 to 35 students. Accordingly, six classes were taken, three of them were chosen randomly as experimental groups and the rest of them were control groups. In purposive sampling, candidates who have similar characteristics are considered and taken as a sample study (Etikan et al., 2016). The three schools out of a total 130 schools were chosen based on the similarity of some important characteristics: socioeconomic background, geographical location, and students' previous aptitude in mathematics and science. These three aspects were considered in sample selection to get an accurate result. In Erbil, people with different socioeconomic backgrounds live in different parts of geographical locations. For example, there is a Golden area where most of the people who live there have a rich economy. In these high socioeconomic status families, most of their children study in the top private schools, or their children have a private teacher for each subject. The aptitude of those students cannot equalize and combine with students who are from lower socioeconomic backgrounds. In the present study, the three schools were chosen in medium socioeconomic areas. Another aspect that was considered in choosing the sample study was, students' previous aptitude in mathematics and science, they were checked by looking at their last year's grades to equalize the groups. The three school administrators were asked to provide me the students' grades for mathematics and science subjects. The students who had on average less than 60% were sorted to a low level, an average of 60% to 80% sorted to a medium level, and an average of over 80% sorted to a high level. Then the groups were redistributed based on the students' level where each group containing an approximately equivalent amount of low, medium, and high-level students. To compare the three schools, the experimental and control groups were compared using a 3x2 ANOVA (school x group) with a pretest. The results show that the schools and classes do not differ in grades, anxiety, and attitudes at the beginning of the study. See [Tables 3, 4, and 5].

Table 3:Pre-Grade, Neither Schools nor Groups Were Significantly Different, and The Interaction Was Not Significant.

ANOVA – Pre-Grade

	df	F	p	η^2p
school	2	1.1667	0.314	0.012
Group	1	0.0226	0.881	0.000
school ✻ Group	2	1.3160	0.271	0.013

Table 4: Pre-Anxiety, Neither Schools nor Groups Were Significantly Different, and The Interaction Was Not Significant.

ANOVA - Pre-Anxiety

	df	F	p	η^2p
Group	1	0.125	0.724	0.001
school	2	2.701	0.070	0.027
Group ✻ school	2	0.490	0.613	0.005

Table 5: Pre-Attitude, Neither Schools Nor Groups Were Significantly Different, and The Interaction Was Not Significant.

ANOVA – Pre-Attitude

	df	F	p	η^2p
Group	1	2.742	0.099	0.014
school	2	1.041	0.355	0.011
Group ✻ school	2	0.318	0.728	0.003

In each of the three sample schools, there were two groups of students. The students at each school were divided into two numerically equivalent groups based on their average mathematics and science grades from the previous year. Consequently, the participants in all three schools represented two large groups whose average mathematics grades resembled one another. Each group in the two large groups contained low, average, and high achievers. In addition, they had the same ages, and they read the same mathematics subjects. One of the groups was designated the experimental group. The other was designated a control group; The groups were chosen randomly. Each group in each of the three schools was called a subgroup. See [Table 6].

Table 6: Sample study

Groups	Subgroups		
Experimental group	A1	A2	A3
Control group	B1	B2	B3

Because of the substantial nature of the period the researcher chose students in eighth grade as a sample study. This grade is the main period when students develop their understanding of mathematics (Hembree, 1990). In this grade's curriculum, there are many mathematics subjects that are basic and foundation for the next four grades, such as functions and equations. Another reason for choosing students at this age is that the most of studies on mathematics anxiety and mathematics attitude have focused on either primary school or adult students; few of them have concentrated on learners in the middle grades (Ashcraft & Kirk, 2001). Accordingly, students in eighth grade were taken as a sample in the present study.

3.4. Instruments

It is not an easy process for the researcher to measure the student's conceptual knowledge. The shortage of having a compact definition for conceptual knowledge makes a difficulty for researchers to evaluate it. Crooks and Alibali (2014) also revealed, there are conflicts between the meanings of conceptual knowledge and the tools used to measure conceptual understanding. Further, the trouble of conceptualizing mathematical knowledge is one of the trunk problems. "…some other troubles with measuring conceptual knowledge in a theoretically grounded way" (Crooks & Alibali, 2014, p. 363). Focusing on the differentiation between conceptual knowledge and procedural knowledge, rather than on attention to measure them is another challenge for lacking precision in measuring conceptual knowledge. However, the trouble of accuracy in measuring conceptual knowledge has not been an obstacle for mathematics researchers, but they are trying and leading to a vast range of tasks to measure conceptual understanding (Crooks & Alibali, 2014).

This study aimed to collect both qualitative and quantitative data to investigate the answer to the four research questions. The quantitative method is defined as "explaining phenomena by collecting numerical data that are analyzed using mathematically based methods in particular statistics" (Aliaga & Gunderson, 2002, p. 1). Richards and Schmidt (2002, p. 436) define the quantitative approach as "procedures that gather data in numerical form". While the qualitative approach tries to obtain the participant's in-depth views, attitudes, feelings, and experiences by using one of the instruments such as an interview (Dawson, 2009). The qualitative approach depicts real-life situations, and it interprets the phenomena based on information mainly earned from participants (Stojanov & Dobrilovic, 2013). Accordingly, the qualitative research approach can provide an in-depth and detailed view of situations and circumstances.

Using more than one tool in data collection makes the study more valid and trustworthy (Cohen et al., 2007). According to Jacob (1998) to reveal new problems and expand ways to solve problems in education the researcher should use a variety of research approaches. Furthermore, Deetz (1996) indicated that using different methods of research helps the researchers to discover various aspects of the phenomena. Therefore, three tools were used in the present study for data collection, interview, experiment, and questionnaire. The main reason behind selecting these three tools was the nature of the research questions (Thomas, 2009). The research tool is chosen based on what the researcher tries to discover (Cavaye, 1996). The interview was used to investigate the first research question that required expressing the mathematics educator's idea. Experiment and questionnaire were utilized to investigate the last three research questions.

3.4.1. Interview

One of the powerful research approaches for exploring the human aspects of a specific practice or type of education is an interview (Stojanov & Dobrilovic, 2013). An interview is a "conversation for gathering information" (Easwaramoorthy & Zarinpoush, 2006, p. 6). "It is an appropriate method when there is a need to collect in-depth information on people's thoughts, and experiences" (Easwaramoorthy & Zarinpoush, 2006, p. 6).

The qualitative interview has become a fundamental tool in mathematics education research, especially after the development of qualitative methodologies (Zazkis &

Hazzan, 1998). In the present research, in-depth and open-ended interviews, or called semi-structured interviews, were applied online. Open-ended questions were utilized to achieve detailed information about perceptions, experiences, and feelings (Patton, 2001). Interviews were conducted to generate rich detail, which is important for understanding mathematics educators' perspectives on conceptual knowledge (Stojanov & Dobrilovic, 2013).

Interview questions were chosen based on the research questions for the study. Then, after a group of scholars in mathematics education approved the interview questions, the final version of the questions was constructed and prepared (see Appendix A).

The interview period lasted approximately three months. The interview stage began on January 1, 2021, and ended on March 20, 2021. Each interview took about 30 minutes that conducted outside of the classroom. All the interviews were transcribed and coded to understand secondary school mathematics teachers' perceptions of conceptual and procedural knowledge in greater depth.

3.4.2. Experiment

In the present study, in addition to interviewing secondary school mathematics teachers in Erbil, an experimental approach was used to discover how teaching mathematics conceptually affects students' achievement in, anxiety about, and attitude toward mathematics. The study tries to prove or disprove its hypotheses statistically (Ross et al., 2005). An experimental approach is a research approach that attempts to reveal the impact of an independent (experimental) variable that the researcher purposely manipulates while holding all other conditions constant. By comparing the experimental group with the control group, the effect of the independent variable will be revealed (Jonassen et al., 2008).

Pretests and posttests were conducted on both groups of students to reveal how teaching mathematics conceptually affect students' achievement (see Appendices B & C). The pretests and posttests were designed to compare groups and to measure changes resulting from a certain treatment (Dimitrov & Rumrill, 2003). The same achievement test was applied to the conceptual and procedural groups. The pretest was conducted at the beginning of the experiment to evaluate students' academic performances. After 5 weeks of teaching, the posttest was applied to reveal how conceptual knowledge impacted students' achievement.

Conceptual knowledge can be measured by scores on problem-solving exams (Mariquit & Luna, 2017). All of the questions on the achievement tests were developed from an eighth-grade curriculum. The pretest covered the topics namely, the meaning of fractions, simplifying fractions, addition and subtraction of fractions, multiplication and division of fractions, absolute value, inequalities, power, decimal, and square roots. The posttest covered curriculum topics that students were taught during the experimental period, namely, equations, solving multi steps inequality, comparing fractions, probability, statistics (mean, median, and mode), and geometry (parallel lines, congruent triangles, isosceles triangles, equilateral triangles, and parallelogram). I chose these topics that were taught during the experimental period, based on the schools' principle that I had to follow the curriculum topics' order. The tests were prepared and applied by means of collaboration between me and the mathematics teachers. There were 25 multiple-choice questions for each of the pretest and the posttest. Students received 1 point for each correct answer and 0 points for each incorrect answer. Therefore, a perfect mark was 25 points. For analysis purposes, the marks were converted to a scale of 100.

3.4.3. Questionnaire

A set of questions that ask participants to answer in a prearranged order is called a questionnaire (Dornyei & Taguchi, 2010; Taylor et al., 2006). According to Johnson and Christensen (2008), there are three types of questionnaires depending on open-ended and closed questions: the qualitative questionnaire, the quantitative questionnaire, and the mixed questionnaire. In the present study, a quantitative questionnaire was utilized. The Mathematics Attitude Scale (MAS) developed by Aiken and Dreger (1961) and the Abbreviated Math Anxiety Scale (AMAS) developed by Hopko et al. (2003) were adopted to determine how conceptual teaching affects students' attitudes toward and anxiety about mathematics, respectively. AMAS and MAS were constructed of 9 questions and 20 questions respectively, all of the questions were multiple-choice questions and have five alternatives (See Appendices F & G). Both AMAS and MAS were translated into the Kurdish language by a Kurdish teacher who is an expert in English-Kurdish translation. At the beginning and the end of the experiment, students in the experimental and control groups were asked to complete MAS and AMAS evaluations. Analyzing the answers that students gave during the pre-experiment and post-experiment periods revealed the effect of conceptual knowledge on students' attitudes and anxiety.

3.5. Research Ethics

The rules and regulations of research ethics were considered carefully in the present study. According to Robson (2011 a & b), ethical principles should be considered in studies that contain participants. In the present study, the principles of research ethics were carefully taken into consideration. The study's ELTE PPK Research Ethics Committee license number is 2020/209. All participations in the present study were voluntary, not physically or psychologically compelled. In the interview, all of them were asked whether they were allowed to have their voice recorded or not; only 18 out of 30 interviewees agreed on recording their sound by the interviewer. Pseudonyms were used for all participants and schools to protect their privacy and confidentiality.

3.6. Evidence of Validity and Reliability

To determine the validity of the interview, a group of experts, which consisted of the researcher's supervisors and mathematics education professors, were invited to review it. After the panel review, comments were collected and considered by the researcher. The panel made a few suggestions for revision; otherwise, the other items in the instrument were found to be commensurate with the research questions for this study.

A study is considered to have good quality if it is approved for internal validity, external validity, and reliability (Guba & Lincoln, 1994). Both internal and external validity were considered in the experiment. Internal validity is the control of external variables to arise the effect of treatment variables accurately (Gall et al., 2003). The characteristics that threaten internal validity were taken into consideration, mainly, using different methods of teaching in the groups, different levels of students' aptitude, socioeconomic background, and the number of students in a classroom. The researcher controlled both procedural and conceptual teaching in the groups by making, on the one hand, a meeting with the mathematics teachers before each class to exchange information on teaching methods. On the other hand, the researcher attended each class to be sure that there is no bias in the teaching method process. Students who were taken as a sample study had almost the equivalent level of aptitude, and they approximately had the same socioeconomic background. Finally, the number of students in the groups was equalized. External validity is the possibility of generalizing the effect of treatment on the populations (Gall et al., 2003). In terms of external validity, the results of this study can only be generalized to students who have characteristics similar to those of the participants in this study (Dimitrov & Rumrill, 2003).

A group of five mathematics education experts validated the achievement test and assessed its content validity. Content validity, “refers to the steps taken to ensure that assessment items reflect the construct they are intended to measure” (Cook & Hatala, 2016, p. 3). The questions were given to a judging panel of subject matter experts (SMEs). The panel made only a few suggestions. Otherwise, the items in the instrument were found to be commensurate with the research questions. Based on the experts’ suggestions, the tests were revised to make them appropriate for use in the study. The measure of the content validity of the whole test called content validity index (CVI) is defined as the average CVR score for all questions in the test (Nikolopoulou, 2022). The CVI for the pretest and posttest were 0.98 and 1 respectively.

The pilot study was conducted to ensure the accuracy and suitability of the data collection instrument. The pilot study aimed to uncover and track any weaknesses before applying the instrument to an entire community (Walsh, 2001). To ensure that the interview questions were appropriate and clear, two mathematics teachers in the Kurdistan region of Iraq were interviewed. The result of the pilot study revealed that all the interview questions were obvious, and 30 minutes allocated for each interview was enough. Meanwhile, a few amendments were made to the questions to be perfect.

To check the reliability of the achievement test, also, a pilot study was applied to 30 students. A difficulty index, discriminant index, and Cronbach’s alpha were used to measure the test’s reliability. The difficulty index shows the level of the questions’ difficulty (Suherman & Sukjaya, 1990). The difficulty indices of the pretest and the posttest were moderate: 45% and 41%, respectively (Suherman & Sukjaya, 1990). While the questions have a good discriminant index if can recognize between low-achieving students and high-achieving students (Suherman & Sukjaya, 1990). The discriminant index for the pretest was .77, and it was .72 for the posttest. These are good levels (Lim, 2007). Each test consisted of 25 questions, yielding good reliability (Cronbach's alpha = .85 and .84).

When studying mathematics anxiety, Hopko et al. (2003) reported excellent reliability values, strong convergent validity, and appropriate internal consistency: Cronbach's alpha = 0.67. The test-retest reliability of AMAS was considered: r = 0.66. In addition, according to Aiken and Dreger (1961), MAS has excellent internal consistency and temporal stability: positive attitude subscale Cronbach's alpha = 0.911, and negative attitude subscale Cronbach's alpha = 0.902.

3.7. The Education System in Kurdistan

In the state school system in Kurdistan, students attend school for four and a half hours per day, six days per week. There are two semesters in an academic year: the first semester lasts from September to January, and the second semester lasts from February to June. There are five mathematics sessions in a week, and each session lasts 40 minutes. The formal method of teaching mathematics in Kurdistan combines conceptual and procedural teaching. However, for a variety of reasons that are explained in the following sections, mathematics teachers generally tend towards procedural teaching.

3.8. Research Focus and Design

Semi-structured interviews were conducted for this study. The interview questions were formulated according to three aims. The first was to reveal mathematics teachers' familiarity with conceptual knowledge and teaching conceptually, in other words, their understanding of conceptual knowledge and the differences between conceptual and procedural teaching. The second was to reveal the importance of teaching mathematics conceptually from the perspectives of mathematics teachers, in other words, why teaching mathematics conceptually is important, and the teaching methods applied by teachers in the classroom. The final perspective was to identify obstacles that mathematics teachers face in teaching conceptual knowledge and how they can be managed. See [Table 7].

In addition, the experimental approach was used to determine the impact of a manipulated variable: teaching for conceptual understanding (Sekaran, 1992). In the experimental (treatment) groups, mathematics instructors focused on teaching for conceptual understanding (see Appendix D). In the control groups, they taught mathematics subjects conventionally (Gürbüz et al., 2010). Conventional teaching in the Kurdistan region of Iraq mostly depends on communicating procedural knowledge. In procedural teaching, the teachers teach how to use the mathematics rules to solve the problem regardless of explaining the relationships between the concepts, and answer about how and why questions in the steps in the problem solving (see Appendix E). Teachers are prepared to teach both conceptual and procedural knowledge. Nonetheless, for many reasons, they focus on procedural understanding; mainly, it is more comfortable for them, the learning materials are available in textbooks, and it does not require them to change their teaching styles (Maryunis, 1989). Teachers in both groups had almost the same amount of teaching experience, which ranged from 8–11 years per person. The duration of the experiment was 5 weeks, beginning on May 16 and ending on June 20, 2021. In each group 25 lessons were taught; the period of each lesson was 40 minutes. In the experimental group's classes, teachers combined conceptual and procedural teaching by focusing on three main aspects: using instructional language carefully and avoiding naked numbers, focusing more on concepts than algorithms and shortcuts, and building connections among concepts. In addition, these teachers concentrated on providing in-depth explanations of the relationships among the steps required to solve a mathematics problem. This teaching method also utilized thinking aloud, one of the most common strategies in metacognition (Moghadam & Fard, 2011). Teaching mathematics conceptually is not a simple process; a certain kind of knowledge is required (Putnam et al., 1992). Nevertheless, I coordinated with mathematics teachers to successfully investigate the experimental process. I worked with mathematics teachers to design and deliver lessons that fostered conceptual knowledge of mathematics. Before each lesson in all groups, I had ten minutes meeting with the mathematics teachers who enrolment in the experimental program, to talk about the lesson and check that everything was well and going in the correct direction. Thereby, I attended most of the classes in both the experimental and control groups to observe the teaching processes and to ensure that the experiment proceeded correctly.

3.9. Implementation of Teaching Conceptually

To teach mathematics conceptually, the teachers in the present study focused on three main aspects: using instructional language carefully and avoiding naked numbers, focusing more on concepts than algorithms and shortcuts, and building connections among concepts (Molina, 2014).

1. Using Instructional Language Carefully and Avoiding Naked Numbers

When teachers use the correct language of mathematics, they protect students from confusion and enable them to obtain a deeper understanding of mathematics. Even a slight deviation in language can cause content errors because mathematics is one of the most accurate disciplines. Language mistakes might occur because of carelessness by mathematics teachers. For example, if a teacher writes the fraction 9/12 and asks students to reduce it, the expected solution is 3/4. Understanding the concept of reduction is different from understanding the concept of simplification, however. These terms have contradictory meanings in relation to the concept of equality between the fractions 9/12 and 3/4.

Teachers should avoid using naked numbers, or numerals without descriptors. It is necessary to connect the idea of measurement and the wider idea of representation. For example, take the problem 6 ÷ 1/2. If students are taught in careless language, this expression may be interpreted as "how many times does 1/2 go into 6?". What does this expression mean? If students get the answer, do they know what it represents? Do they know why the result is larger than the original instead of smaller? What if the teacher asks, "How many halves are there in 6"? in that case, students realize that the answer is 12 halves, not only the naked number 12.

2. Focusing More on Concepts Than Algorithms and Shortcuts

Algorithms and shortcuts are beneficial only when they help conceptual understanding rather than hindering it. Teachers must provide students with detailed steps about how to solve a mathematics problem. They can explain why those steps happen and connect the concepts with the process.

Understanding mathematics procedurally rather than conceptually makes it harder for students to absorb more complicated subjects. When using a shortcut method, the problem-solving notion remains fuzzy for students. Accordingly, each step in the problem-solving process must be included. This inclusion is fundamental for a deeper understanding of what exactly happens when an equation is solved.

3. Building Connections Among Concepts

Finding connections among mathematical concepts and ideas can be used to develop mathematics pedagogy. Recognizing connections is the basis for a deeper understanding of mathematical concepts. For example, take the concept of average. The main point here is a profound understanding of the concept of multiplication. Generally, multiplication is defined as repeated addition. The crucial missing point in this definition is the repetition of groups of equal sizes. Therefore, multiplication can be defined as the repeated addition of groups of equal sizes. In 2 × 5, for example, the explanation could be 5 + 5. This equation represents 2 groups, each of which has 5 units. To achieve a deep understanding of the concept of average, it is necessary to relate it to the concept of division. Students will recognize that division means equal distribution and that the average is defined as an equal redistribution. This conceptual definition could not be obtained without making a connection to multiplication and division.

3.10. If Students Understand Mathematics Conceptually, They Can Answer These Kinds of Questions

1. Why can we multiply a number by zero, but we cannot divide by zero?

Let is we can divide a number by zero. Division and multiplication are inverse operations. Dividing a number by 5 gives an answer that can be multiplied by 5 to get the original number. Likewise, dividing a number by zero should give a result that can be multiplied by zero to get the original number.

For example:

$20 \div 5 = x$ means $5 \times x = 20$ then $x = 4$

$20 \div 0 = x$ means $0 \times x = 20$ there is not any number multiplied by zero equal to 20, then $20 \div 0$ is undefined means there is no way to describe that value.

2. If students understand the meaning of inverse and multiplication, they have to be able to answer the question "Why in the division of fractions, we multiply the first fraction by the inverse of the second fraction?"

The students have to know that multiplication and division are inverse operations. Therefore, dividing by a number is the same as multiplying by its inverse (the inverse of 5 is 1/5, and the inverse of 6/7 is 7/6).

3. If a student understands the meaning of division conceptually, he/she has to know how to answer this question "How does dividing an integer number by a fraction get a bigger number?"

Imagine we have 10 packs of sugar, if we divide by 1/2, means splitting each pack into half. Then we get the result 20! this 20 is not packs but it is 20 half packs. Therefore, the student should know that this result is not an integer number.

4. In Exponents in Algebra $(x^2)^3 = x^{2\times3} = x^6, but\ how$?

$(x^2)^3 = x^2 \times x^2 \times x^2 = x \times x \times x \times x \times x \times x = x^6$.

5. In Exponents in Algebra $(xy)^2 = x^2y^2, but\ How$?

$(xy)^2 = xy \times xy = xx \times yy = x^2y^2$.

6. The circumference of a circle is $\boldsymbol{\pi} \times \boldsymbol{diameter}, but\ what\ is\ \pi$?

$\pi = 3.14, but\ what\ is\ this\ number?\ \pi = \frac{\text{circumference of the circle}}{\text{diameter of the circle}}$.

7. In the power subject $x^0 = 1,\ how$?

$1 = \frac{x}{x} = x^{1-1} = x^0\ except\ x = 0$.

3.11. Research Paradigm

The term "Paradigm" or "widely conceived research methodologies" is mentioned in the previous literature (see for example, Neuman, 2009). A paradigm is defined as a "loose collection of logically held together assumptions, concepts, and propositions that orientate thinking and research" (Bogdan & Biklen, 1982, p. 30). And paradigm is an "identification of the underlying basis that is used to construct a scientific investigation" (Krauss, 2005, p. 759).

There are four components in the paradigm: ontology, epistemology, methodology, and methods (Rehman & Alharthi, 2016). Ontology is "the nature of our beliefs about reality"

(Richards, 2003, p. 33), which means the researchers' assumptions about the existence of reality. Epistemology is "the branch of philosophy that studies the nature of knowledge and the process by which knowledge is acquired and validated" (Gall et al., 2003, p. 13). The methodology studies the data production techniques. It is defined as "an articulated, theoretically informed approach to the production of data" (Ellen, 1984, p. 9). While the method is an approach that the researcher follows to collect and analyze data, such as an interview or questionnaire (Rehman & Alharthi, 2016).

There are five common paradigms in the literature: positivism, constructivism, postpositivism, transformativism, and pragmatism (Guba & Lincoln, 1994; Mertens, 2019; Rehman & Alharthi, 2016). While the most popular paradigms in social science are positivism and constructivism (Guba & Lincoln, 1994; Mertens, 2019).

In the positivism paradigm, knowledge is revealed through measuring and observing the situation directly. By examining the component parts of the situation, the facts are discovered (Rehman & Alharthi, 2016). The positivism paradigm depends on the same assumptions studied the natural world for studies in the social world. positivist research tries to investigate the general rules that represent constant relations among variables based on measure and experiment (Creswell & Creswell, 2017).

The constructivism paradigm also called the interpretivism paradigm (Grix, 2004), is the knowledge that is established as a result of attaching meanings to the studied aspect (Cousin, 2002). The constructivism paradigm employs methods that generate qualitative data mainly. The constructivism paradigm generally supports the qualitative method of research and assumes that research should be conducted through interaction between the researcher and the participants (Mertens, 2019). In the constructivist paradigm, the reality is limited to time, context, and groups in specific situations (Guba & Lincoln, 1994)

Postpsoitivisim contains both positivism and interpretivism (Grix, 2004; Healy & Perry, 2000). In postpositivism, there is doubt about making generalizable laws by researchers that can be applied to human actions and behavior (Mertens, 2019). In postpositivism, objectivity and generalizability are concentrated on, but probability instead of certainty is based on claiming (Mertens, 2019). It is also known as realism or neo-postpositivism (Manicas & Second, 1992). Both the positivism and postpositivism paradigms are followed mainly in quantitative research.

Since social science has started to deal with more complex problems, modern views to know and conduct social research have arisen. These include the transformative paradigm and pragmatism. The transformative paradigm is generally associated with participatory action research in that various version of reality is recognized. While mixed-method studies associate with the pragmatism paradigm (Creswell & Clark, 2017).

The pragmatic paradigm guided the research design of the present dissertation. The philosophy of applying this paradigm depends on the research questions because, in the pragmatic paradigm, reality is continually interpreted. Both positivism and constructivism paradigms are combined in a pragmatic paradigm that is used in mixed-method research. Pragmatists believe that the best research method is one that answers the research questions accurately and successfully. In this paradigm, ontology and epistemology focus on debating how the social world can be known. The researchers do not commit to any one system of philosophy (Johnson & Onwuegbuzie, 2004). It develops pluralistic approaches on how to investigate the research problem (Patton, 2002). Different forms of data collection are used by the researcher to investigate the necessary knowledge to get the answer to the research question (Creswell, 2014).

3.12. Chapter Summary

This chapter explained how all the data of the study were collected scientifically and who were the target group in the study (the population and the sample study). Furthermore, the chapter detailed the instruments that the researcher used to collect the data, namely, interview, experiment, and questionnaire. Research ethics is a main aspect that was taken into consideration in this study, and was clarified in the present chapter. Next, the research focus and design, thereby, the implementation of teaching conceptually were illustrated. The chapter was finalized with shed light on the research paradigm and chapter summary.

Chapter Four

4. Data Analysis and Results

This chapter explains how data were analyzed to get the answer to the four research questions. The chapter consists of two main sections. The first section shows how the interview data were analyzed based on the aforementioned three main perspectives to investigate the first research question. The second section details how the experimental data and the survey data were analyzed.

4.1. Interview

After the data collection stage, the researcher transcribed all audio recordings and read them several times for accuracy. For participants who did not consent to be recorded, the interviewer took notes during the interview. The interviews were transcribed on the same day that they took place (ideally directly after each interview) to reduce recall bias. In a process called condensation, the text was shortened while maintaining its core meaning (Erlingsson & Brysiewicz, 2017). To facilitate the analytical process, coding techniques were used to identify and record underlying ideas in the data. The coding process can be used to clarify, structure, and develop deeper meanings from the interview conversations. According to Erlingsson and Brysiewicz (2017, p. 2), “a code can be thought of as a label; a name that most exactly describes what this particular condensed meaning unit is about, usually one or two words long.” Deductive coding was used in the present study to focus on the research questions. Also known as concept-driven coding, the process of deductive coding begins with predefined codes, which are then applied to the new qualitative data (Medelyan, 2021). The next step was to categorize and group the codes to make sense of the data. Codes that are related in content or context can be grouped to make a category (Erlingsson & Brysiewicz, 2017). Next, three main themes were identified based on the aforementioned three main perspectives (Erlingsson & Brysiewicz, 2017). Then, different ideas and themes are related to each other to answer the research questions (Rubin & Rubin, 1995). See [Table 7].

Table 7:Data Analysis Classifications

Familiarity with conceptual knowledge	**Meaning of conceptual knowledge**	**Familiarity without any explanation:** Around 20% of participants (six interviewees) were familiar with the term "conceptual knowledge."
		Familiarity with the researcher's explanation: Seventy percent of participants (21 interviewees) recognized the meaning of conceptual knowledge after the researcher's explanation.
	Differences between conceptual and procedural teaching	**Teaching conceptually, teaching procedurally:** Overall, 63.3% of participants (19 interviewees) named clear differences between the two teaching methods.
Perspectives on teaching mathematics conceptually	**Perspectives on the importance of teaching mathematics conceptually**	**Teaching conceptually is not important:** Overall, 6.6% of participants (two interviewees) believed that teaching mathematics conceptually was not important.
		Teaching conceptually is important: Overall, 93.3% of participants (28 interviewees) believed that teaching mathematics conceptually was important.
	Teaching methods used by teachers in the classroom	**Teaching procedurally:** Overall, 73.3% of participants (22 interviewees) only taught mathematics procedurally.
		Teaching conceptually and procedurally: Only 26.6% of participants (eight interviewees) combined conceptual and procedural teaching.

<table>
<tr><td colspan="2">Factors needed for conceptual teaching</td><td>- More time
- Training course for teachers
- Reducing the amount of students' curriculum
- Teaching method in Kurdistan</td></tr>
<tr><td rowspan="2">Obstacles to teaching conceptual knowledge</td><td>Obstacles</td><td>- Insufficient time
- Insufficient knowledge among teachers.
- Pressure by school administrators and supervisors to complete the curriculum during the academic year.
- Some mathematics teachers believed that conceptual teaching complicates mathematics for students.
- Many students only want to pass their mathematics course rather than develop a deep understanding of the topic.</td></tr>
<tr><td>Potential solutions</td><td>- Increase the duration and weekly frequency of mathematics classes.
- Hold open training courses for mathematics teachers.
- Encourage school administrators and supervisors to not only focus on completing the curriculum but also on achieving a better understanding.
- Foster mathematics communities to exchange information.
- Make contentment for mathematics teachers that teaching mathematics conceptually is not a waste of time and does not make mathematics more complicated for students, by academic debate.</td></tr>
</table>

Most participants (80%) were not familiar with the term “conceptual knowledge.” For example, one participant said, “I do not know exactly what you mean by “conceptual knowledge.” However, 70% of participants had some understanding of the term when the researcher offered some explanation. One participant said, “After your clarification, now I know exactly what you mean by teaching mathematics conceptually and conceptual knowledge.” In addition, 63.3% of participants were able to differentiate between teaching conceptually and procedurally.

While some interviewees were familiar with teaching mathematics conceptually, they did not apply it to their classroom teaching. Most participants (93.3%) believed that teaching mathematics conceptually was important and necessary for students to develop a better understanding of mathematics. One interviewee stated, “if we want to teach mathematics better in the classroom, we have to start by teaching conceptually first then explaining procedurally.” Interviewees did not doubt that imparting conceptual knowledge in addition to procedural knowledge would increase students’ mathematics performance. When the researcher asked participants why they did not use this practice in the classroom, many answered that they did not have enough time to teach mathematics in this manner. Moreover, two participants believed that teaching mathematics conceptually would make the subject more complicated for students.

Interviewees identified several prerequisites for teaching conceptual knowledge in mathematics. Most participants (73.3%) believed that teaching conceptually was time-consuming and would result in them not being able to complete the curriculum. One participant said, “I do not want to waste my time because I have to finish the curriculum by the end of the academic year, otherwise, I will be blamed by the administration and the supervisors of the school.” Thus, they proposed that a reduction in the curriculum content would make teaching conceptually more feasible. One interviewee said, “If we need to teach conceptually, curtailing the curriculum will be needed. Because I do not have enough time to teach all the curriculum conceptually” Another stated, “Without reducing the curriculum, it is impossible to apply conceptual teaching in the classroom.” Professional development was also a necessary factor for mathematics teachers to familiarize themselves with teaching conceptually, as they must have up-to-date knowledge of teaching methods. One participant said, “We need training courses if we want to teach mathematics conceptually because there are a lot of questions that should be discussed.” Finally, the same teaching methods should be used by mathematics teachers. Participants believed that the education directors, heads of schools, and school supervisors should require mathematics to be taught conceptually. One interviewee said, “Of course, if there is no rule that forces me to teach conceptually, I will use the easiest and the quickest method of teaching.” Another asked, “Why do I have to teach conceptually, which consumes more time and energy compared to teaching procedurally?”

In addition, participants mentioned obstacles in teaching conceptually, mainly a lack of time. The interviewees believed that they had a very limited time to teach mathematics and thus could not teach it at a deep level. One participant said, “Teaching conceptually needs more explanation. Therefore, it consumes more time. According to our school's rule, we do not have enough time for that teaching.” In addition, mathematics teachers were not entirely familiar with teaching conceptually. Another obstacle is that both school administrators and supervisors are very strict about completing the curriculum during the academic year. This leads many teachers to teach mathematics procedurally rather than conceptually. One respondent said, “Completing the curriculum and students’ grades are the core aspects for school administrators and mathematics inspectors, so I should focus on these.” Some mathematics educators believed that teaching conceptually was not necessary because it complicated mathematics for students. One participant asked, “Why do I have to make mathematics more complicated for students by teaching conceptually?” Another respondent said, “When I try to explain mathematics to students too much, they become confused.” Finally, many students only aim to pass their mathematics course; they do not want to understand mathematics too deeply. Instead, they focus on procedures for solving mathematics problems to pass the exam.

4.1.1. Sample of Interviews Transcript

- **Participant A Reacted to the Interview Questions as Follow**

> In conceptual teaching, the teacher should provide real examples, that exist in the student’s real environment. This example will help the students to remember the mathematics subject in the future easily. While, in procedural teaching, the mathematics teacher only provides the rules to students and teaches them how to use these rules to solve a mathematics problem.

The language that mathematics teacher use in the classroom is very important. The mathematics teacher should use accurate language in the teaching process. For example, when we talk about minus and negative, we have to be aware to not confuse our students. In addition, when we talk about division and fractions we have to be aware because most of the students do not understand the meaning of these two terms conceptually and they are confused.

I was not entirely familiar with these two terms conceptual teaching and procedural teaching, but based on your explanation for these two terms, I have been teaching conceptually because I think it is essential for students' understanding. That is true it takes more time than teaching procedurally, but I believe that even if I cannot finish the whole curriculum at the end of the academic year, it is worth following conceptual teaching. I recommend the Ministry of Education in the Kurdistan region of Iraq review the teaching method that mathematics teachers follow in the classrooms.

Opening professional courses for us is essential to develop our efficiency in teaching because we need to update our information regarding teaching and understanding mathematics by sharing our knowledge, questions, and struggles with experts and between us.

Time is an obstacle for teaching conceptually. According to my experience, conceptual teaching needs more explanation, and it needs to provide more examples that lead to consuming more time which I do not have it. In addition, conceptual teaching needs more afford than procedural teaching, this is why the majority of mathematics teachers prefer procedural teaching rather than

> conceptual teaching, while it is very obvious that teaching conceptually is much more fruitful than procedural teaching.
>
> Regarding the curriculum, some amendments are necessary. The curriculum is too much for teaching conceptually because it needs a lot of time. Therefore, some subjects should be deleted from the curriculum and some subjects should be reduced. For example, the Probability subject should be moved to the 9^{th}-grade curriculum totally because this subject is explained in grade 9^{th} in detail.

This participant answered the first question of the interview partially, he thought that conceptual teaching means that mathematics teachers provide students real examples. However procedural teaching means providing the students mathematical rules and teaching them how to use them which is totally correct. This interviewee mentioned an essential point that the mathematics teachers' language that uses should be taken into consideration otherwise it makes a problem for learners' understanding because of the sensitivity of mathematics subject. Furthermore, he is very interested in teaching mathematics conceptually, but he has some difficulties with this teaching, mainly, a lack of information, and not enough time.

- **Participant B Reacted to the Interview Questions as Follow**

> I have not heard these two terms, but I think these two terms relate to the students' happiness with mathematics or it relates with the relationship between mathematics teachers and the students. But, after your explanation, now I understand what you mean by conceptual teaching and procedural teaching. I think not only me, but all mathematics teachers should believe that teaching mathematics conceptually is essential for students' understanding of mathematics successfully.

> In the teacher guidebook, should have more explanation about conceptual teaching. The teacher guidebook is one of the main sources that mathematics teachers follow in the teaching process. Therefore, the meaning of the term conceptual teaching and the steps of teaching conceptually should be detailed in the teacher guidebook.
>
> One of the main problems in mathematics subject is the method of teaching that the teacher follow it. In my view, teaching is art before science. Especially in teaching mathematics, if the teacher can attract the student's attention with his/her writing, drawing, and nice explaining, then the learning process will be easier. Mathematics is one of the boring subjects for the students, it becomes more boring with using unpleasant teaching methods. Thus, anything that helps students to better understand mathematics has to be taken into consideration.
>
> Time is the problem, we do not have enough time. Teaching mathematics conceptually is taking more time than procedurally. Therefore, if I have to teach conceptually, the class period and the frequent of math class should be increased.

This interviewee did not know what conceptual and procedural teaching are, but only she guessed that these two terms relate to the student's happiness in mathematics class. However, after the interviewer's explanation, she realized what these two terms mean. She proposed that mathematics teacher's guidebook should be developed more, and the instruction about conceptual teaching should be added there. Then show talked about the importance of the teaching method that the teacher follows in their classroom. Many obstacles for conceptual teaching are mentioned in this interview but the most crucial one is that they do not have enough time for teaching in that way.

- **Participant C Reacted to the Interview Questions as Follow**

I have not heard these two terms yet. But after your explanation, I understand exactly what you mean by conceptual teaching and procedural teaching. I would like to say that I am very happy that your study is about teaching conceptually because I believe that neglecting conceptual knowledge by educators is the main problem in teaching and learning mathematics. All mathematics teachers who I know, teach mathematics procedurally rather than conceptually because it is easer and quicker. This procedural teaching of mathematics makes mathematics classes boring classes for students.

I teach conceptually sometimes in my classroom because I believe that conceptual teaching is extremely necessary if there are no obstacles. Meanwhile, we need to use procedural teaching as well, especially in problem-solving and after students understand conceptually.

There are many obstacles for teaching conceptually. First, the academic level of the teachers is not quite enough for this teaching. Second, I do not have enough time to teach conceptually. Third, the curriculum is too much for this kind of teaching.

Students be happy if the teacher makes a consultation with them in problem-solving, for example, the teacher can ask the students what do you think about this problem. How do you understand it? How can we start to solve it? Why do we use this approach for solving? Are there any different ways for solving the problem?

> I believe that all the subjects in the curriculum are necessary and it is not possible to delete a part of it, rather, the class period or the frequency of the class in a week should be increased to teach conceptually successfully.
>
> I believe that this is the Ministry of Education's responsibility in Kurdistan to work on it. The Ministry of Education should encourage mathematics teachers to follow conceptual teaching rather than procedural teaching, by opening professional courses and motivating them to make a community to make an academic debate.

This interviewee believes that teaching conceptually is essential for students' higher achievement. However, there are some obstacles to this teaching that stakeholders have to find solutions for them, especially the Ministry of Education in Kurdistan. He also talked about the importance of both conceptual and procedural teaching. He believes that a balance between these two teaching methods by mathematics teachers is necessary.

- **Participant D Reacted to the Interview Questions as Follow**

> I am not quite familiar with these two terms. But I heard from one of my friends who studied postgraduate in mathematics education. I remember that the meanings of conceptual teaching and procedural teaching exist in the context of her dissertation. However, I do not know a lot about them.
>
> I think that teaching mathematics conceptually is necessary for students' better understanding of mathematics subjects. But according to my experience, only a few mathematics teachers follow conceptual teaching, otherwise, most of them teach in a procedural way because it is easier and quicker. I know that teaching mathematics conceptually more powerful than procedural teaching, but my teaching process is going with procedural teaching without any problems and all of the students, school administration,

> and mathematics supervisors agree on my education, then why do I make difficulties for myself with conceptual teaching?
>
> There are many problems with conceptual teaching that have to be solved, mainly, the number of students in a classroom is very high which affects the teaching process and the student's understanding. The explanation in the student's curriculum should be more detailed which helps the students to make a self-study beside the teacher's explanation. We do not have to forget that mathematics teachers should be more prepared for conceptual teaching by giving them instruction, making a professional community, and opening developing courses. I think mathematics teachers should be prepared for conceptual teaching at the university level. Because study in university is basic for developing their knowledge and it is essential for their future teaching in a conceptual way successfully.

This interviewee heard the terms conceptual and procedural teaching in her friend's dissertation. She believes that teaching conceptually is necessary for students' deeper understanding of mathematics. However, she said that with teaching procedurally she has not had any problems at the same time all of the students and the school administration are satisfied. Then, why do I have to teach conceptually, and why do I have to make a problem for myself? She proposed that teacher preparation courses should be taken more into consideration at the university level. Because mathematics teachers need more knowledge on conceptual teaching.

4.2. Experiment and Survey

Different techniques were utilized to statistically analyze the data and thereby answer the last three research questions. Descriptive statistics—namely, standard deviation and mean—were used to describe students' performance in, anxiety about, and attitude toward mathematics (Andamon & Tan, 2018).

Descriptive statistics analysis shows that the mean score for Post-Grade in the experimental group increased more than in the control group compared to their Pre-Grade average. The mean score for the experimental group is 69.0 in Pre-Grade, but this mean score reached 72.1 in Post-Grade. However, in the control group, there is only a 0.5 difference between Pre-Grade and Post-Grade mean scores. The difference in mean scores between Pre-Attitude and Post-attitude in the treatment group is 14.5, which is much higher than the difference in the mean scores between Pre-Attitude and Post-Attitude in the control group 3.6. Meanwhile, in the experimental group, the student's anxiety was reduced according to Post-Anxiety test scores (mean = 26.5) compared to Pre-Anxiety (mean = 31.6). Nevertheless, in the control group, the mean score of students' anxiety slightly decreased from 31.8 to 30.6. See [Table 8].

Table 7: Descriptive statistics for experimental and control group measures

Measure	**Group**	**N**	**Mean (%)**	**SD**
Pre-Grade	experimental	100	69.0	15.89
	control	100	69.4	14.51
Post-Grade	experimental	100	72.1	16.42
	control	100	69.9	14.79
Pre-Attitude	experimental	100	55.1	16.83
	control	100	59.0	16.55
Post-Attitude	experimental	100	69.6	12.90
	control	100	62.6	13.17
Pre-Anxiety	experimental	100	31.6	3.55
	control	100	31.8	3.77
Post-Anxiety	experimental	100	26.5	3.74
	control	100	30.6	3.58

To determine whether the developmental sessions produced any improvement in mathematical achievement, grade scores were analyzed using mixed analyses of variance (2 groups × 2 genders × 2 measurements). The difference between the mathematical achievement pretest and posttest scores was significant: $F(1,196) = 18.48$, $p < .001$, partial eta squared = 0.086). Students achieved higher scores in the posttest than in the pretest. The difference between genders was not significant: $F(1,196) = 0.04$, $p = 0.83$, partial eta squared = 0.000. Likewise, the interaction between group and gender was not significant: $F(1,196) = 0.79$, $p = 0.375$, partial eta squared = 0.004. In contrast, the interaction of group × gender × measurement was significant: $F(1,196) = 4.52$, $p = 0.035$, partial eta squared = 0.023. Based on Tukey multiple comparisons, the experimental group's mathematical abilities improved significantly more for girls than boys. There was no change in the control group throughout the 5 weeks. See [Figure 2].

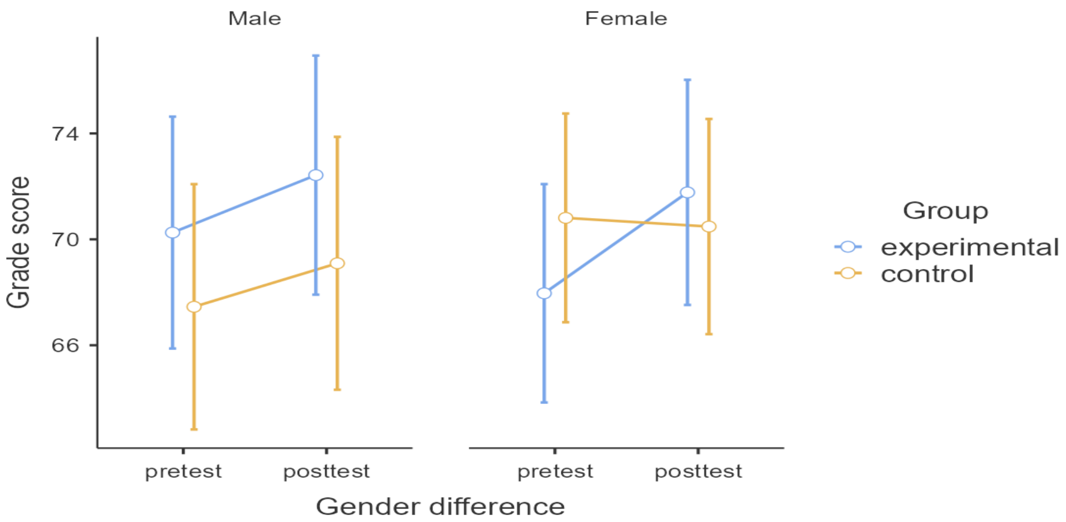

Figure 2: The difference between grade scores for male and female students in experimental and control groups. The interaction of group × gender × measurement was significant, the experimental group's mathematical abilities improved significantly more for girls than boys. Error bars show the 95% confidence intervals in both diagrams.

Mixed ANOVA measurements showed a statistically significant difference between the control group and the experimental group in terms of pretest and posttest scores: $p = 0.007$. The participants in the treatment group achieved a higher score than the participants in the control group in the achievement test. See [Figure 3].

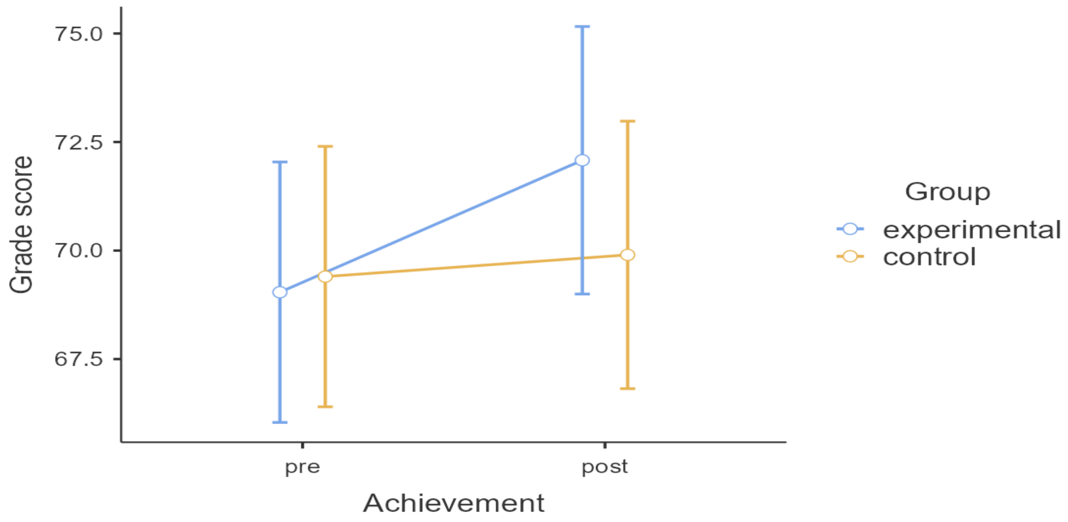

Figure 3: The difference between grade scores for students in experimental and control groups. The participants in the treatment group achieved higher mathematics scores than the control group. Error bars show the 95% confidence intervals in the diagram.

According to the mixed ANOVA measurements, there was a statistically significant difference between the attitudes of the experimental and control groups: $F(1,198) = 149$, $p < 0.001$, partial eta squared = 0.429. See [Figure. 4]. The experimental group overperformed the control group in developing a positive attitude toward mathematics. Likewise, there was a statistically significant difference between the anxiety levels of the experimental and control groups: $F(1,198) = 117$, $p < 0,001$, partial eta squared = 0.372. See [Figure. 5]. This statistical analysis shows that the students' anxiety in the treatment group decreased more than the students' anxiety in the control group after the experiment period. In addition, it shows that the girls had higher anxiety than the boys: $F(1,196) = 4.33$, $p < 0.001$, partial eta squared = 0.022.

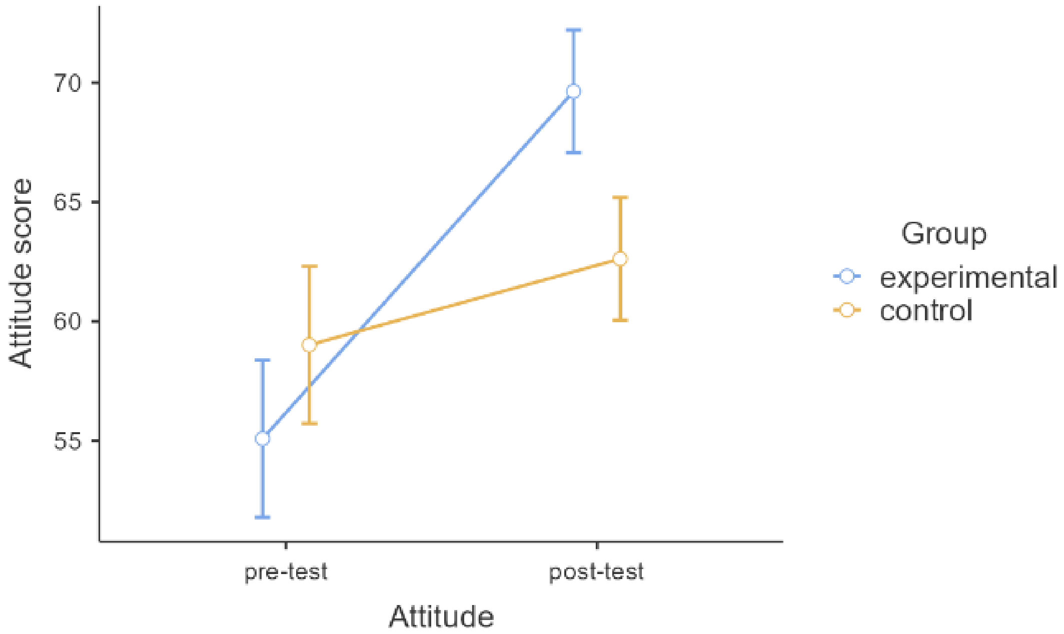

Figure 4: Estimated marginal means for positivity of students' attitude toward mathematics. The attitude toward mathematics is more positive for the experimental group than the control group. Error bars show the 95% confidence intervals in the diagram.

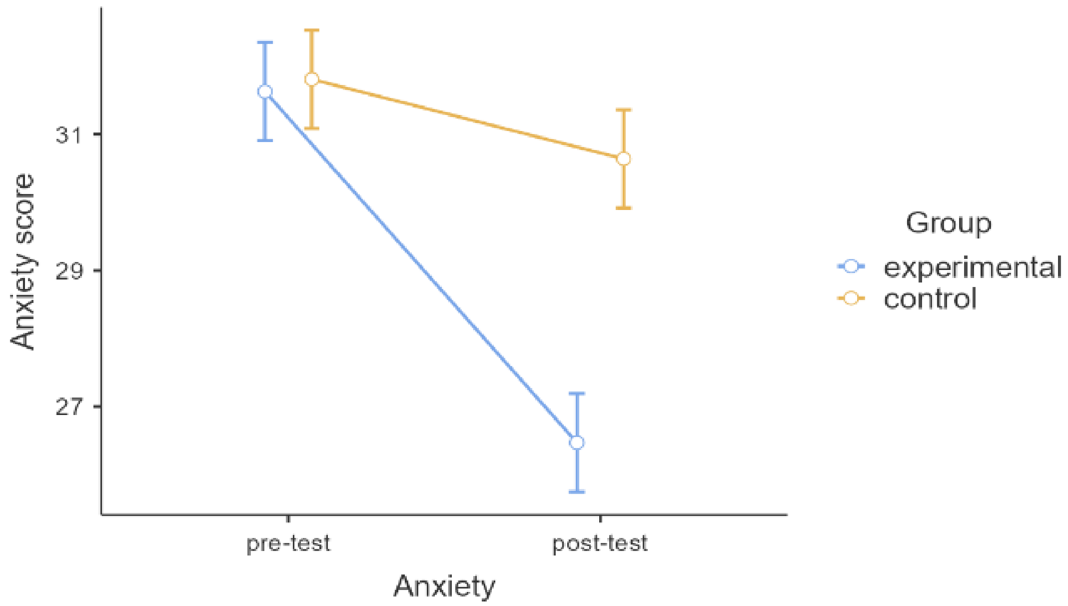

Figure 5: Estimated marginal means for students' anxiety. The control group had higher anxiety levels than the experimental group. Error bars show the 95% confidence intervals in the diagram.

A high F value represents an effect variance that exceeds the error variance by a large amount. In the present analysis, the *F* value was high, which aligns with prior educational research. For example, in a study on the consistency and variability of learning strategies in different university courses, four university courses were taken. That study's authors found that "Lack of regulation was reported less frequently in the Private Law and Criminal Law courses compared to the other courses in both studies (F (1, 84) = 86.19, p < 0.001, and F (1, 62) = 56.06, p < 0.001" (Vermetten et al., 1999, p. 13).

The ratio of partial eta squared was elevated. Nevertheless, according to many researchers, a high value of partial eta squared in educational research is expected. For example, according to Sechrest and Yeaton, (1982), it is possible for the sum partial eta squared to exceed 1.00. Le Poire and Yoshimura (1999) used mixed ANOVA to reveal the effect of different independent variables on a single dependent variable. Partial eta squared was 0.74 for the effects of communication. For communication by manipulation, partial eta squared was 0.83.

From that perspective, the three null hypotheses in the present study are rejected. The following substitutes replace them: "There is a statistically significant difference between the control group and the experimental group in terms of students' achievement," "There is a statistically significant difference between the control group and the experimental group in terms of the degree to which students' anxiety decreased," and "There is a statistically significant difference between the control group and the experimental group in terms of the degree to which students' positive attitude toward mathematics improved."

4.2. Chapter Summary

This chapter explained how the data were analyzed to investigate the four main research questions. In this chapter, all collected data were transformed into information by using specific statistical analysis. Two major sections make up the chapter. The first section showed how the interview data were analyzed based on the aforementioned three main perspectives to investigate the first research question. The analysis of the experimental data and the questionnaire was described in depth in the second part.

Chapter Five

5. Discussion and Conclusion

This chapter tries to answer the research questions by interpreting the results of the study and connecting them with the previous literature findings. In order to connect the study's findings to earlier research, the chapter has gone over how the findings fit into the body of knowledge and whether they confirm or refute preexisting findings. The results of the study's implications for future research directions or potential applications in the real world may have also been covered in the chapter. The study is concluded by summarizing the key points that were found in the present dissertation. Thereby, some recommendations for concerned people are provided and finalized with the limitation of the study.

The goal of this research is to investigate the views of math teachers on the value of conceptual teaching and the challenges they confront. Additionally, it investigates how conceptual understanding among secondary school students in Iraq's Kurdistan region affects their performance in, anxiety about, and attitude toward mathematics. Its objective is to disseminate knowledge and provide advanced mathematics instruction.

The following research questions provide a framework for the study investigation:

1. What is the importance of conceptual knowledge in teaching mathematics for students from mathematics teachers' perspectives?

   This question has four sub-questions:

   a. What is mathematics teachers' familiarity with conceptual understanding?
   b. What are mathematics teachers' perspectives on teaching mathematics conceptually?
   c. What do mathematics teachers need to teach conceptually?
   d. What are the obstacles that mathematics teachers face when teaching mathematics conceptually?
2. Does teaching mathematics conceptually affect students' achievement?
3. Does teaching mathematics conceptually affect students' anxiety?

4. Does teaching mathematics conceptually affect students' attitudes?

5.1. Interview

Over the past few decades, there has been an increase in the number of studies on conceptual understanding. They emphasize students' understanding of mathematical concepts and their ability to solve mathematical problems (Star, 2005). Likewise, participants in the present study believed that imparting both conceptual and procedural knowledge was necessary for students to better understand mathematics. One participant noted, "It will be very helpful if all mathematics teachers teach conceptually and procedurally." This comment refers to the many useful aspects of conceptual knowledge. Firstly, conceptual knowledge can help students to evaluate the most suitable procedure for a specific mathematical problem (Carr et al., 1994; Garofalo & Lester, 1985). Secondly, it provides more flexibility in problem-solving, as students with adequate conceptual knowledge can generalize procedures to a new problem (Baroody et al., 2007; Blöte, 2000; Rittle-Johnson et al., 2001). Thirdly, it can be used to check the truth of a solution after the problem has been solved (Garofalo & Lester, 1985). Fourth, conceptual knowledge can give students greater confidence when they are confronted with different mathematical problems (Carr et al., 1994; Garofalo & Lester, 1985; Korn, 2014; Schneider & Stern, 2010). Finally, with structured and organized knowledge, students can relate information beyond isolated facts or automatic procedures (Bransford et al., 2000). All these points are made to encourage mathematics researchers and educators to focus on conceptual knowledge.

The present study shows that the participants had some background in teaching conceptually. However, they did not try to teach conceptually in their classrooms. Only 20% of interviewees were able to define conceptual knowledge without any explanation from the researcher. When prompted, they remembered what they had learned about conceptual knowledge. This means that mathematics teachers in Kurdistan generally focus on teaching procedurally in their classrooms despite having some background in conceptual teaching.

Regarding teaching methods, only eight participants stated that they combined conceptual and procedural teaching. One of them said, "My students are very happy because I explain mathematics to them very clearly and deeply.” It seems that these students appreciate this teaching method. Another interviewee said, “It is true that I am a bit more tired than usual with teaching conceptually besides procedurally, but my students are comfortable because they can understand real mathematics.” The other participants used the procedural approach to mathematics teaching. Three participants believed that teaching mathematics in depth and explaining it in terms of relationships made mathematics too complicated for students; as a result, the students would dislike mathematics class even more. One interviewee said, “I explain mathematics rules to the students, and I teach them how to use those rules in solving mathematics problems. Why would I need to make the mathematics class more complicated by giving them deeper explanations?” Another stated, “We do not have problems with teaching procedurally, and my students’ grades are reasonable.” In addition, seven interviewees believed that teaching conceptually was only necessary for some subjects in mathematics. One said, “Some of the subjects in mathematics need conceptual teaching, but some others do not need it. For example, some very pure mathematics subjects can only be explained procedurally.” Six of the interviewees believed that teaching conceptually not only depended on teaching methods but also on the curriculum, school system, and school environment. One participant stated, “The four columns—the teacher, students, school system, and curriculum—are necessary to support the conceptual teaching of mathematics.”

In terms of solutions, interviewees mentioned that increasing the number of mathematics classes per week and class duration were needed to apply conceptual teaching. Interviewees believed that increasing the duration of mathematics classes from 40 minutes to 70 minutes and the number of mathematics classes from five to six classes per week should be considered. In addition, training sessions that focus on up-to-date teaching methods should be offered to mathematics teachers. One participant stated, “Mathematics teachers need to participate in training courses to develop their knowledge about teaching conceptual knowledge.” Moreover, it is crucial for mathematics teachers to have their own communities in which to exchange knowledge and discuss teaching problems with mathematics experts. Furthermore, school administrators and supervisors should not only focus on completing the curriculum but also on teaching quality and ensuring students’ understanding. Through academic debate, mathematics teachers must be persuaded that teaching mathematics conceptually is neither a waste of time nor makes mathematics more complicated for students. Finally, both students and teachers should be encouraged to focus on conceptual understanding alongside procedural understanding by formulating exam questions that require students to have conceptual knowledge to correctly answer.

Despite a noticeable shift in focus toward conceptual knowledge among researchers and educators, participants in this study mentioned many obstacles to teaching mathematics conceptually. First, conceptual knowledge can be implicit or explicit, which means that it might be not verbalizable (Goldin-Meadow et al., 1993). Only around half of the participants could differentiate between teaching mathematics conceptually and procedurally. Some of them confused the two teaching approaches, while others provided ambiguous answers (e.g., "By teaching conceptually, the mathematics teacher will connect the subject with our daily lives, while, by teaching procedurally, the mathematics teacher only focuses on pure mathematics."). Seven interviewees did not want to provide any explanation and said that they did not remember at the moment. The participants also believed that, for some advanced mathematics subjects, the topic cannot be explained in depth; instead, it can only be explained procedurally. This is consistent with previous studies that indicated that conceptual and procedural knowledge cannot be easily differentiated because they are so deeply intertwined (Baroody & Lai, 2007; Crooks & Alibali, 2014; Long, 2005; Star, 2005).

Another obstacle to conceptual teaching is pressure from supervisors and school administrators to complete the curriculum and increase students' pass rates. Thus, the focus is on quantity rather than quality in students' understanding of mathematics. According to Zakaria et al. (2010), school administrators encourage mathematics teachers to concentrate on student achievement in exams and on completing the curriculum regardless of students' satisfaction with mathematics courses or depth of understanding. Therefore, most mathematics assessments traditionally depend on students' ability to procedurally manipulate knowledge. Assessment tools focus on procedural knowledge rather than both conceptual and procedural knowledge (De Zeeuw et al., 2013).

Insufficient understanding of the nature and structure of mathematical knowledge is another reason why teachers focus on procedural knowledge rather than conceptual knowledge (Hallett et al., 2012; Lin et al., 2013). In this study, interviewees believed that mathematics teachers did not have enough knowledge to teach all mathematics subjects conceptually. Therefore, they proposed training courses for mathematics teachers and the development of communities for academic discussion with support from mathematics experts.

Procedural knowledge has become standard knowledge for solving mathematics problems. For example, students are graded on exams based on the number of correct answers (Rittle-Johnson & Siegler, 1998). This is consistent with the results of the present study, as some participants believed that teaching conceptually was not necessary because assessment tools are based on procedural knowledge. One of the participants said, “I tell my students, mathematics rules and how to use them for problem-solving. My students’ grades are reasonable, and we do not have a problem with procedural knowledge.”

Finally, some mathematics teachers believe that prioritizing conceptual knowledge is time-consuming compared to procedural knowledge because this requires more explanation and a deeper understanding of the topic (Baroody & Lai, 2007; Crooks & Alibali, 2014). Participants in this study confirmed that teaching conceptually is time-consuming, which is difficult to manage. For example, one interviewee said, “It is not easy to teach mathematics conceptually in the classroom within a 40-minute class period.” However, according to Andrew (2019), it is time-consuming not to prioritize procedural knowledge over conceptual knowledge in mathematics teaching because students spend significant amounts of time not understanding what they are working on; as a result, mathematics courses become unpleasant and boring. Andrew provided two main reasons for this. Firstly, a better understanding of mathematics reduces the time that students spend being confused. If students do not understand key concepts, they struggle to remember rules and procedures. Secondly, students with a good understanding of mathematics require less practice. If mathematics teachers apply a conceptual approach to teaching first and a procedural approach later, students do not require much practice to solve problems (Andrew, 2019).

5.2. Experiment

This study shows that, in terms of mathematics achievement, there is a statistically significant difference between the experimental and control groups. The teachers in the treatment group followed lesson plans focused on conceptual understanding. Teachers in the control group followed the principles of conventional teaching. Teaching mathematics conceptually led students in the experimental group to achieve higher mathematics exam scores than students in the control group. The experimental group's mean achievement posttest score was much higher than its mean pretest score. In the control group, only a slight difference was recorded between the pretest and posttest scores. This finding indicates that conceptual teaching affected mathematics achievement in the experimental group. This finding is consistent with Khoule et al.'s (2017) study, which showed that the students in a conceptual group performed better on conceptual and procedural quizzes than those in a procedural group. Furthermore, students in the conceptual group were more able to reason logically, formulate solutions, and understand mathematics flexibly.

Teaching mathematics conceptually helped students achieve higher scores on mathematics exams because these students learned mathematics based on relations and concepts rather than procedures. Relational understanding helped the students remember mathematics rules more easily, and it provided them with the ability to adapt their knowledge to solve new mathematics problems. Students who possess relational understanding will be active in finding new areas in which to apply mathematics—for instance, they might apply it to the roots of trees that extend in all directions (Skemp, 1976). In contrast, it was difficult for students in the control group to apply what they learned in the classroom on the posttest. In particular, they failed to answer questions with different contexts from those that they solved during class. This finding is consistent with Zakaria et al.'s (2010) study, in which a significant relationship between conceptual knowledge and mathematics achievement was recorded. Conceptual teaching helps students to better achieve the learning process goal (Hurrell, 2021).

This study also found that teaching mathematics conceptually had a different impact on different genders (see [Figure. 2]). Female students in the experimental group achieved higher exam scores than male students. This result contradicts the findings of Hyde et al. (1990), who found no statistically significant difference in mathematics achievement between male and female students. This might suggest that female students have more mathematics anxiety than male ones (Beesdo et al., 2009). It was found that teaching for conceptual understanding reduces female students' anxiety more than it does male students'. Teaching for conceptual understanding reduces female students' anxiety more than it does male students'. As a result, female students achieved higher scores than male ones on the mathematics test. Students with lower levels of mathematics anxiety tend to have a stronger understanding of arithmetic than students with higher levels of mathematics anxiety (Price, 2015). However, girls have higher mathematical anxiety and this coupled with higher intrinsic motivation, can improve their math performance (Wang et al. 2015).

Students in the experimental group learned to self-monitor by asking themselves many questions during the problem-solving process. For instance, they asked, “How can I start to solve this problem?” “What is the relationship among these steps?” and “What is another method to solve this problem?”. These kinds of questions help learners improve their metacognitive ability, which helps them to understand mathematics in a deeper and more meaningful way (Ilyas & Basir, 2019; Kramarski et al., 2002; Salehi, 2002). This finding is consistent with the findings of previous studies. For example, Amin and Sukestiyarno’s (2015) study found that metacognitive knowledge plays an active role in improving students’ achievement. According to Grant (2014), students’ ability to monitor themselves during the learning process can increase their ability to solve problems.

Reducing mathematics anxiety helps students achieve better results in mathematics exams (Furner, 2019). The correlation between students’ mathematics anxiety and their performance in mathematics courses was negative and very high (Hembree, 1990). To increase students’ mathematics achievement, educators must work on reducing their anxiety. Lau et al. (2022) found in their study that to reduce students' anxiety, teachers have to use an appropriate teaching method and have confidence in teaching mathematics. According to Ashcraft and Moore (2009), “Math anxiety is a significant impediment to math achievement” (p. 197). Additionally, Jameson (2014) found that mathematics anxiety negatively impacts mathematics achievement in both children and adults. The findings in the present study support Ashcraft and Kirk’s (2001) study, which revealed that reducing students’ mathematics anxiety allows them to concentrate their attention more on mathematical tasks that develop their competence. The present study is consistent with the finding of a study conducted by Casty et al. (2021) on mathematics anxiety, attitude, and performance among secondary school students, that both students' anxiety about and attitude toward mathematics affected their performance and achievement in mathematics. Students with high anxiety about and negative attitude toward mathematics had less confidence to carry out mathematics tasks. Accordingly, for students’ better achievement in mathematics, students’ level of anxiety and attitude toward mathematics should be considered and moderated by using an appropriate teaching method (Casty et al., 2021; Richland et al., 2020).

Students' mathematics anxiety increases unless teachers teach conceptually. According to a theory by Skemp (1971), procedural teaching increases students’ mathematics anxiety. The present study found a negative relationship between conceptual knowledge and participants’ mathematics anxiety. After learning mathematics conceptually for 5 weeks, the mathematics anxiety of students in the experimental group was reduced. In terms of anxiety reduction, the treatment group outperformed the control group. The experimental group’s mean score on the anxiety posttest dropped remarkably compared to its pretest results. In the control group, however, the mean score in the pretest was 31.8, a number that only dropped to 30.6 on the posttest. These results indicate that teaching mathematics conceptually helps students to reduce their anxiety which leads to higher achievement.

Students in school are taught how to memorize mathematics rules and how to use those rules to solve mathematics problems, but they do not work on comprehending the explanations of the skills they learn (Rossnan, 2006). In a study conducted by Khoule, et al. (2017), algebra teachers in community colleges mostly depended on procedural methods. Focusing on memorization techniques rather than conceptual understanding encourages students' mathematics anxiety (Newstead, 1998). According to a study by Curtain-Phillips (1999), students’ anxiety has increased due to the fact that teachers do not cater to various learning styles. Mathematics anxiety can develop in response to unsupportive teaching styles (Webb, 2017). Therefore, to overcome students’ mathematics anxiety, educators must find different methods of teaching mathematics (Rossnan, 2006). For example, they can teach conceptual understanding in addition to teaching conventionally. Teachers can start with visual models and finalize the lesson with a symbolic model that expresses abstract concepts in symbols (Ketterlin-Geller, 2007).

Compared to students in the experimental group, students in the control group found it harder to remember mathematical rules during the posttest. Many students in the control group asked, “Teacher, could you explain to me which rule I have to use to answer question number *x*?” It seems that they had high levels of anxiety because teaching mathematics procedurally increased their anxiety rather than overcoming it (Khoule, et al., 2017). Moreover, high levels of anxiety make it difficult to remember mathematical rules (Rayner et al., 2009).

The present study statistically revealed that the conceptual method of teaching can overcome students’ anxiety about mathematics. The traditional method, also known as the procedural method, cannot reduce students’ anxiety because it depends on memorizing rules. The students had high levels of anxiety because they were taught mathematics procedurally which increased their anxiety rather than overcoming it (Khoule et al., 2017). Moreover, high levels of anxiety make it difficult for them to remember mathematical rules (Rayner et al., 2009).

For students to be successful in learning mathematics, teachers must take both cognitive and affective aspects into consideration (McLeod, 1994). In the present study, the effect of a conceptual teaching method on students’ attitudes toward mathematics revealed that the attitudes of participants in the experimental group improved more than the attitudes of those in the control group. The experimental group’s mean posttest score for positive attitude was higher than its mean pretest score, but this was not the case for the control group. According to Ashcraft and Kirk (2001), maintaining a positive attitude toward mathematics helps to increase students’ competence in mathematics courses. This finding is consistent with a study conducted by Jennison and Beswick (2010) on the effect of an interactive teaching method on students' attitudes toward and performance in mathematics. The study’s results revealed that the majority of participants increased their confidence and their positive attitudes toward mathematics by the end of the study. In addition, a study conducted by Zamir et al. (2022) on determining students' attitudes and achievements through problem-based learning, revealed that mathematics attitude is considered as a critical element in the process of mathematics learning. Confidence in learning mathematics and mathematics motivation had a significant role in the students’ attitudes toward problem-based learning. Therefore, there is a strong association between students' attitudes toward mathematics and their performance in this subject. Students with a positive attitude toward mathematics have higher performance than whom have negative attitudes toward it (Naungayan, 2022; Segarra & Julià, 2021).

In summary, most of the interviewees in the present study believed that teaching both conceptual and procedural knowledge was necessary for students to better understand mathematics. this finding is consistent with Turner et al.’s (2002) study, which showed that using more than one teaching approach by the teachers helps the students to develop their confidence and achievement. However, the participants in the present study preferred procedural teaching for several reasons, the main ones being the pressure that they are under to complete the curriculum in an academic year and ensure a high pass rate among students and the fact that they regard teaching procedural knowledge as being easier.

Moreover, the results show that students will earn a higher score in mathematics if they participate in thinking and exploring rather than merely learning mathematics rules mechanically (Khoule et al., 2017). Teaching mathematics conceptually built confidence among this study's participants. It decreased their mathematics anxiety, and it helped them create the confidence needed to absorb new mathematical knowledge. To summarize, teaching mathematics conceptually not only improves students' achievement in mathematics but also reduces their mathematics anxiety and improves their positive attitudes toward this subject.

5.3. Conclusion

The study's main findings are detailed in this section, which also explains how they connect to the study's research questions and hypothesis. This last section provides an interpretation of the findings, brings the investigation to a close, and leaves the reader with a lasting impression of the study.

Mathematics does not consist of a collection of isolated facts and algorithms; rather, it is a web of interconnected elements (Nik Pa, 2003). Likewise, there is a relationship between conceptual knowledge and procedural knowledge; gains in conceptual knowledge lead to increases in procedural knowledge (Lauritzen, 2012; Rittle-Johnson et al., 2001; Rittle-Johnson et al., 2015). Therefore, this study investigates the importance of conceptual teaching in addition to procedural teaching in mathematics from the perspectives of mathematics teachers in Kurdistan, with the aim of disseminating the results. The study focuses on mathematics teachers' perspectives on teaching mathematics conceptually, the conditions needed to teach conceptually, and the obstacles that they face in teaching mathematics conceptually. Furthermore, it investigated how teaching mathematics conceptually impacts students’ achievement in, anxiety about, and attitudes toward mathematics.

The results revealed that most interviewees believed that teaching conceptual knowledge alongside procedural knowledge was crucial for students to have a better understanding of mathematics (Nahdi & Jatisunda, 2020). This finding is consistent with previous studies. For example, the National Governors Association Center for Best Practices and Council of Chief State School Officers (2010) stated that, by focusing on conceptual knowledge in mathematics teaching, students would gain a deeper understanding of mathematics and that information would be retained for a longer period of time. To improve learning quality and student achievement, it is vital to help students to understand mathematics conceptually. Once students gain conceptual knowledge, they can assess the suitable procedure to use in a specific mathematical problem (Brownell, 1945; Schneider & Stern, 2010).

However, only a few interviewees in the present study combined procedural and conceptual teaching in their classes. Participants saw teaching conceptually as time-consuming because it requires more explanation and a deeper understanding of the topic. Others believe that teaching conceptually was not necessary because it made mathematics more complicated for students. These reasons discouraged mathematics teachers from teaching conceptually.

Moreover, in this study experimental approach was utilized. The students in both groups (experimental and control group) have similar experiences and the same academic background. The students in both groups are provided the same mathematics activities during the five weeks of the experiment. The only difference between them was the teaching method. In the experimental group, conceptual teaching was focused on, while the control group was taught traditionally. The teaching process in both groups was observed during the experiment by the researcher.

The results from the experiment revealed that there are statistically significant relationships between teaching for conceptual understanding and the three aforementioned variables. Compared to the control group, the experimental group performed better in a mathematics exam. This result indicates that participants in the experimental group outperformed the control group in mathematics achievement, reduced their mathematics anxiety, and improved their attitudes toward mathematics.

5.4. Recommendations and Suggestions

In this section, the researcher highlights ideas to push for more research, make suggestions for methodological or data analysis enhancements, or offer recommendations for practitioners who wish to apply the findings in their work. The recommendations and suggestions section is crucial since it provides readers with insightful information on how the research might be improved or expanded. They also encourage additional study and contribute to the field's continuing development.

Based on the results of the present study, some recommendations were formulated for mathematics teachers, school administrators, supervisors, education directors, and the Ministry of Education, Kurdistan of Iraq.

The conventional teaching method should be revised to match the skills that students need to be productive (Rossnan, 2006). Students need practical mathematics classes, and they should be involved in thinking and analyzing rather than merely learning the rules and applying them (Curtain-Phillips, 1999). When learning mathematics, metacognition must be emphasized as much as cognition because it is one of the main elements in students' achievement. Therefore, it is recommended that mathematics teaching focuses on both conceptual knowledge and procedural knowledge since conceptual teaching is key to a better understanding of mathematics among students.

Through academic debate, education directors, school administrators, and supervisors should persuade mathematics teachers that teaching mathematics conceptually is not a waste of time and does not make mathematics more complicated for students.

Increasing the frequency of mathematics classes from five to six classes per week is recommended to allow teachers to have more time to explain mathematics. And exam questions should be formulated in a way that requires students to have a conceptual understanding of the topic; this would motivate both students and teachers to focus on conceptual understanding alongside procedural understanding.

Educators should encourage students to acquire the ability to confront mathematics problems (Furner & Berman, 2003). Students are often afraid of mathematics, which generates anxiety more than other disciplines (Shore, 2005). According to one study, almost two-thirds of adults in the United States had a deep fear of mathematics (Burns, 1998).

However, anxiety can be controlled and reduced (Tobias, 1993). To reduce mathematics anxiety, educators must focus less on speed tests (Reys et al., 1995). Instead, they have to focus more on adopting new teaching methods that are conformable to mathematics (Zamir et al., 2022). Additionally, mathematics teachers must make mathematics an enjoyable subject by using a meaningful method of teaching, and they must also explain the importance of mathematics to everyday life and to students' future careers (Cruikshank & Sheffield, 1992). Furthermore, Woolfolk (1995), provided mathematics teachers some points to avoid students from mathematics anxiety, mainly, all instructions should be clear for students, avoid the pressure of tests by deleting unnecessary pressure parts, and promote the student positive behavior.

I have the same belief as Taylor and Brooks (1986): both basic concepts and correct procedures are important when solving mathematics problems. Therefore, I recommend that mathematics teachers should use multiple teaching methods to build connections between abstract thought and conceptual learning (Hurrell, 2021).

5.5. Limitation of the Study

To provide a more precise and insightful interpretation of the findings, researchers must be aware of and open about the limitations of their study. Researchers can better comprehend their research's advantages and disadvantages, as well as identify areas that may need further investigation or development, by being open and honest about the study's limits. The limitation of the study is characteristics that affect the generalization and interpretation of the result of the study (Price & Murnan, 2004).

This study like the previous studies has some limitations. One limitation of this study is that the study focused only on students in secondary schools in the center of the city, of Erbil. Therefore, I do not know what would be the result if I use schools in villages for this research. Another limitation of this study is I do not use other variables that are related to mathematics, for example, special ability and memory. Finally, there is a question so far whether there is a difference between 13-14 years old students who are at the beginning of the formal operational stage (Piaget's formal operational stages) and 16-17 years old students (Kuhn, 1979).

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Appendices

Appendix A

The questions of the semi-structured interviews:

Name of the teacher:

Date:

1. Gender: ☐ Male ☐ Female
2. How many years do you have experience in teaching mathematics?
   ☐ 1-5 ☐ 6-10 ☐ 11-15 ☐ 16- over
3. What is the academic certificate that you have?
4. How do you define conceptual knowledge in mathematics teaching?
5. What is the difference between conceptual understanding and procedural understanding?
6. Is teaching mathematics conceptually necessary? Why?
7. Which approach conceptually or procedurally do you use for teaching mathematics in your classroom? Why?
8. Do you advise mathematics teachers to focus on conceptual knowledge as much as procedural knowledge in teaching mathematics?
9. In your perspective, what do you need to teach mathematics conceptually?
10. What are the difficulties of teaching mathematics conceptually?
11. How would be managed these difficulties?
12. Who can manage these difficulties?
13. Do you have any more ideas on this title that you want to add?

Appendix B

Pre-Test	Kurdistan Region of Iraq, Erbil City	State Schools
Mathematics	2021	Grade 8th

Q1/ Choose the correct answer: (25 Degree for all, only one degree for each question).

1. The simplest form of this fraction $\frac{16}{24}$ is.
   A/ $\frac{8}{12}$ B/ $\frac{2}{3}$ C/ $\frac{3}{2}$ D/ $\frac{4}{6}$
2. The simplest fraction form of this decimal 1.125 is.
   A/ $\frac{9}{8}$ B/$\frac{45}{40}$ C/ $\frac{8}{9}$ D/$1\frac{5}{40}$
3. The decimal form of this fraction $-\frac{37}{20}$ is.
   A/1.85 B/-0.85 C/-1.58 D/-1.85
4. The result of $|7| + |-14|$ is.
   A/ 7 B/21 C/ -7 D/-21

Q2/ Compare. Use $<$ *or* $>$ *or* $=$.

1. $|-4| \cdots 3$
2. $|3-5| \cdots |5| - |13|$
3. $\left|-1\frac{2}{3}\right| \cdots \frac{3}{2}$

Q3/ If $y = -\frac{3}{9}$, which is NOT equal to y?

A/ $\frac{-1}{3}$ B/ $-\frac{1}{3}$ C/ -$\left(\frac{-1}{3}\right)$ D/ -$\left|\frac{-1}{-3}\right|$

Q4/ What is the correct answer to the following?

1. $-8.01 - 9.02$=
   A/17.03 B/-1.01 C/ 1.01 D/ -17.03
2. $\frac{4}{9} + \frac{7}{15} =$
   A/ $\frac{41}{45}$ B/$\frac{11}{24}$ C/ $\frac{11}{15}$ D$\frac{42}{45}$
3. $3\frac{1}{2} + \left(-7\frac{4}{5}\right)$=
   A/$4\frac{3}{10}$ B/ $-4\frac{3}{10}$ C/ $-10\frac{13}{10}$ D/$4\frac{3}{3}$
4. $8.25 - \frac{5}{16}$ =
   A/ 8.5625 B/ -7.9375 C/ 7.9375 D/ 3.25
5. $10.71 \div (-0.7)$ =
   A/15.3 B/ -15.3 C/ -7.497 D/ -15.5
6. $6\frac{3}{7}\left(\frac{7}{8}\right)$=
   A/$6\frac{35}{56}$ B/$6\frac{21}{56}$ C/ $5\frac{35}{56}$ D/ $6\frac{10}{15}$

Q5/ The value of $\left(\frac{1}{2}\right)^4 - 3^2$ is.

A/ $-\frac{142}{16}$ B/$\frac{143}{16}$ C/ $-\frac{144}{16}$ D/ $-\frac{143}{16}$

Q6/ Multiply or Divide. Write the product as one power.

7. $\frac{4^7}{4^3}$
8. $y^8 \times y^{-8}$
9. $(7^4)^3$

Q7/ The standard notation of 4.05×10^{-6} is.

A/ 0.00000405 B/ 0.00000045 C/4050000 D/ - 0.00000405

Q8/ The scientific notation of 0.000000615 is.

A/615$\times 10^{-6}$ B/615$\times 10^{9}$ C/615$\times 10^{-9}$ D/ 615$\times 10^{6}$

Q9/ The result of $5\left(\sqrt{225} - 10\right)$ is.

A/125 B/5 C/-25 D/25

Q10/ Which is NOT equivalent to $3 \times 3 \times 3 \times 3 \times 3 \times 3$?

A/ 729 B/ 18^2 C/ 3^6 D/ 9^3

Q11/ Awaz has a pot containing $\frac{3}{4}$ liters of liquid, he needs to put them in cups of capacity $\frac{1}{8}$ liters. How many cups does he need?

A/ 6 cups B/ $\frac{2}{4}$ C/ 8 cups D/ $\frac{2}{32}$

Q12/ India's population is approximately 1.08×10^9. How do you write this in standard form?

A/ 1080000000 B/ 1080000 C/ 108000 D/ 180000000

Q13/ In Lana's refrigerator 5 grape juice cans, she drinks $\frac{1}{4}$ can per day. For how many days are these cans enough?

A/ 20 days B/ 80 days C/ 5 days D/ 4 days

Appendix C

Post-Test	**Kurdistan Region of Iraq, Erbil City**	**State Schools**
Mathematics	**2021**	**Grade 8^{th}**

Q1/ Choose the correct answer: (25 Degree for all, only one degree for each question).

1. The solution of $3x - 6 > 18$ is?
   A/ $x > 21$ B/$x > 8$ C/$x < 6$ D/$x < 8$

2. On her last three science tests, Mariam got 84,96 and 88. What grade does she need to get on her next test to get an average of 90 on these 4 tests?
   A/90 B/95 C/92 D/ 100

3. Ahmed and Dara together have 36 posters. Ahmed has double what Dara has, how many posters does each one have?
   A/ Ahmed 24, Dara 12 B/ Ahmed 12, Dara 24 C/ Ahmed 18, Dara 18 D/ Ahmed 15, Dara 20

4. What is the value of $(5-3)^{-3} + (845-3)^0$?
   A/ $\frac{1}{8}$ B/ 1 C/ $1\frac{1}{8}$ D/$\frac{7}{8}$

5. Which number is not a solution for the inequality $n - 7 < 1$?
   A/ 2 B/ 4 C/ 6 D/ 8

6. In order to have the 600000 D he needs for a bike, Dlir plans to save an amount of money each week for the next 15 weeks. What is the minimum amount that Dlir has to save each week in order to reach his goal?
   A/ 60000 B/66000 C/30000 D/ 40000

7. The solution of $\frac{5}{6}x + \frac{1}{2} < \frac{2}{3} + \frac{1}{6}x$.
   A/ $x < \frac{1}{4}$ B/$x > \frac{1}{4}$ C/ $x < 4$ D/ $x > 4$

8. Which one is not the same as this fraction $\frac{13}{15}$?
   A/ $\frac{39}{50}$ B/ $\frac{65}{75}$ C/ $\frac{91}{105}$ D/ $\frac{26}{30}$

9. The solution of $\frac{2}{3} + \frac{5}{7}$ is?
   A/$\frac{7}{10}$ B/$\frac{29}{21}$ C/$\frac{7}{21}$ D/$\frac{10}{21}$

10. The solution of $\frac{41}{48} - \frac{5}{6}$ is?
    A/ $\frac{1}{48}$ B/ $\frac{36}{48}$ C/$\frac{36}{42}$ D/ $\frac{1}{6}$

11. If $9 + 3x = 2y$, which of the following is a solution of this equation?
    A/ $\frac{9+3y}{2} = x$ B/ $x = \frac{2}{3}$y-9 C/ $x = \frac{2}{3}y - 3$ D/ $x = 2y - 3$

12. Naveen spins the spinner at right. What is the probability that the spinner will land on the number 4?

A/ 4 B/ $\frac{1}{4}$ C/ 2 D/ $\frac{1}{2}$

13. Which percent best shows the probability that Amir will randomly draw a non-odd number from five cards numbered 2,4,6,8, and 10?
A/ 75% B/25% C/50% D/100%

14. Azad made 26 of the 32 free throws he attempted. Which percent is closest to the experimental probability that he will make his next free throw?
A/ 50% B/60% C/ 70% D/80%

15. What is the mean of this data set: 272, 276, 281, 279, 276?
A/ 276.8 B/ 267.8 C/ 282.1 D/ 285

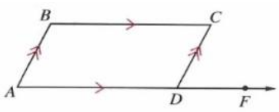
16. In which data set are the mean, median, and mode all the same number?
A/ 6,2,5,4,3,4,1 B/ 4,2,2,1,3,2,3 C/ 2,3,7,3,8,3,2 D/ 4,3,4,3,4,6,4

17. The median of this set 92, 88, 65, 68, 76, 90, 84, 88, 93, 89 is?
A/ 83 B/ 90 C/ 88 D/ 93

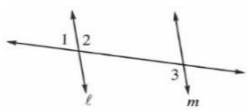
18. A bag contains 25 blue balls, 22 brown balls, and 68 red balls. What is the probability of randomly selecting a blue ball from the bag?
A/ $\frac{115}{25}$ B/ $\frac{22}{115}$ C/ $\frac{5}{23}$ D/other

19. A bag contains 13 yellow balls, 5 black balls, and 17 red balls. What is the percentage probability of randomly selecting a yellow ball from the bag?
A/ 20% B/52% C/ 68% D/13%

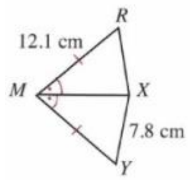
20. Which expression is true for this data set 15, 18, 13, 15, 16, 14?
A/ Mean<mode B/ Median> mean C/ Median=mean D/ Median= mode



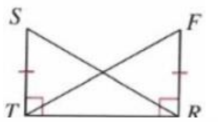
21. $m\,\widehat{BCD} = (x+50)^\circ$; $m\,\widehat{CDA} = (3x+20)^\circ$ Find $m\,\widehat{BAD}$
A/ 15° B/ 27.5° C/ 65° D/ 77.5°



22. What is the relation between $\hat{1}$ and $\hat{3}$?
A/ supplementary B/ alt-int C/ same-side Int D/ given vertically opposite angles

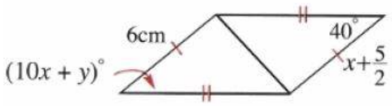


23. What is the perimeter of the polygon MRXY?
A/ 29.9 cm B/ 39.8 cm C/ 49.8 cm D/ 59.8 cm



24. What postulate you can use to prove $\Delta STR \cong \Delta FRT$?
A/ ASA B/ SSS C/HS D/SAS

25. What is the value of y in the adjacent figure?
A/ 5 B/ 20 C/ 35 D/ 10



Appendix D

A Lesson plan sample for teaching conceptual understanding.

Subject: Fraction

Tools: whiteboard, color marker, learning cards

Time: 40 minutes

Objective: The objective of this lesson was to help the students to investigate a deep understanding of the concept of fraction by providing examples in real life and explaining the relation of fraction with division and ratio, also explaining the meaning of simplifying fractions.

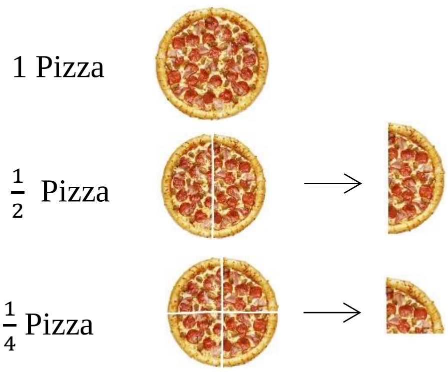
Methodology: Conceptual teaching was utilized, that focused on understanding the concepts and relations.

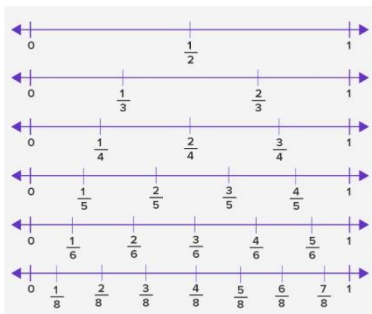
Stages: There were six stages in this lesson plan.

Stage 1 (4 minutes): In this stage, the meaning of fraction was provided, “Is a numerical quantity that represents a part of the whole”. In addition, the writing of fraction and the name of each part, numerator, and denominator is explained.

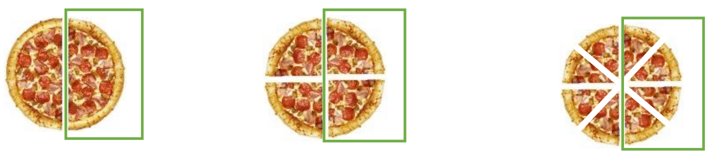
Stage 2 (10 minutes): Some real examples to explain the notion of fraction were provided in the classroom. In this stage, the teacher spends more time deepening in explanation and providing more examples, visually and orally.

For Example:





The same technique was used to explain the notion of simplifying and equality of fractions.

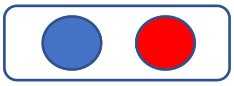


$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Stage 3 (5 minutes): In this stage, the subjects that have a relation with fraction were explained. Here the concept of division and ratio were explained and compared with fraction.

For example:

In ratio $\frac{1}{2}$ means one of two, such as one ball is red in two balls.



In ratio $\frac{2}{3}$ means two of three, such as two are boys out of three students.

More examples were provided to them with graphs, $\frac{5}{10}, \frac{4}{5}, \frac{7}{9}, \ldots$

Also, the relation between fraction and division was explained by the teacher. Fraction is a single number, while division is an operation between two numbers. For example, $9 \div 3, 48 \div 12, 18 \div 9$.

Stage 4 (6 minutes): After explaining the notion of fraction, simplifying, and equality of fractions conceptually, the teacher talked about the fraction formulas and how to use them for solving a mathematical problem.

$$\text{Fraction} = \frac{x}{y} \ \forall\, x, y \in Z \ ; \ \text{for example, } \frac{3}{5}, \frac{7}{12}, \frac{72}{115}, \ldots\text{etc.}$$

In simplifying fraction, we have to find a number that is both of numerator and denominator divided by it, and we will continue until there is not any more division.

$$\frac{4 \div 2}{8 \div 2} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$\frac{36}{123}, \frac{225}{510}, \frac{50}{310}, \frac{28}{84}, \frac{121}{286}, \ldots\text{etc.}$$

Stage 5 (10 minutes): The questions in the textbook were solved by the students with the teacher's help in this stage.

Stage 6 (5 minutes): Evaluate students' understanding of the concept of fraction by asking them some questions, such as, what is fraction? What is different among fraction, division, and ratios? Why does the value of fraction not change if we divide or multiply both the numerator and denominator by the same number? In addition, the participants were evaluated during the 5th stage.

Appendix E

A Lesson plan sample for teaching procedurally (traditional teaching)

Subject: Fraction

Tools: whiteboard, color marker, learning cards

Time: 40 minutes

Objective: The objective of this lesson was to help the students to understand the meaning of fraction mathematically, know the formulas and how to use them in solving problems, and explain how to simplify a fraction.

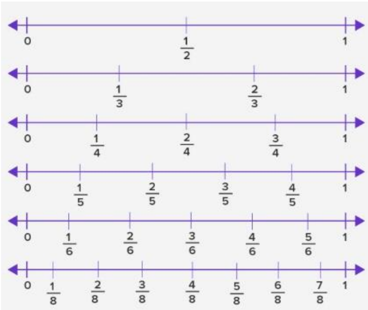
Methodology: Procedural teaching was utilized, that focus on how to use the rules to solve a fraction problem.

Stages: There were five stages in this lesson plan.

Stage 1 (5 minutes): In this stage, the meaning of fraction was provided, "Is a numerical quantity that represents a part of the whole". In addition, the writing of fraction and the name of each part, numerator, and denominator were explained.

Stage 2 (5 minutes): Some examples to explain the meaning of fraction was provided in the classroom. In this stage, the teacher spends less time on explanations, and providing some examples.

For Example:



Stage 3 (10 minutes): The teacher talked about fraction formulas and how to use them for solving a mathematics problem.

$$\text{Fraction} = \frac{x}{y}\ \forall\, x, y\ \in Z\ ;\ \text{for example, } \frac{3}{5}, \frac{7}{12}, \frac{72}{115}, \ldots\text{etc.}$$

In simplifying fraction, we have to find a number that is both of numerator and denominator divided by it, and we will continue until there is not any more division.

$$\frac{4 \div 2}{8 \div 2} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$\frac{36}{123}, \frac{225}{510}, \frac{50}{310}, \frac{28}{84}, \frac{121}{286}, \ldots\text{etc.}$$

Stage 4 (15 minutes): The questions in the textbook were solved by the students with the teacher's help.

Stage 5 (5 minutes): Evaluate students' understanding of fractions by asking them some questions, such as, what is fraction? How do simplify fraction? Why does the value of fraction not change if we divide or multiply both the numerator and denominator by the same number? In addition, the participants are evaluated during the 4th stage.

Appendix F

Abbreviated Math Anxiety Scale (AMAS) developed by Hopko et al. (2003)

Please write your name in the upper right-hand corner. Read the following statements carefully and decide how anxious you would be (how anxious you would feel) in the following situations? Please cross the correct number with an X.

1 - Not at all   2 - A little   3 - Medium   4 - Quite   5 - Very

For example, if you feel that you are not bothered at all when you have to answer, mark 1, and if you really do, mark 5.

There is no right or wrong solution, we want to know your feelings in each situation.

1. Use the tables at the back of the mathematics textbook.	1	2	3	4	5
2. To think about the mathematics test due in 1 day.	1	2	3	4	5
3. Watch the teacher solve an algebraic equation on the board.	1	2	3	4	5
4. Take an exam in a mathematics course.	1	2	3	4	5
5. Get homework with complicated tasks, which should be solved by the next hour.	1	2	3	4	5
6. Listen to a lecture in mathematics class.	1	2	3	4	5
7. Listen to another student explain a mathematical formula.	1	2	3	4	5
8. Write a pamphlet in math class.	1	2	3	4	5
9. Start a new chapter in a mathematics book.	1	2	3	4	5

Appendix G

Mathematics Attitudes Scale (MAS) developed by Aiken and Dreger (1961).

Please write your name in the upper right-hand corner. Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to show, on a five-point scale, the extent of agreement between the feeling expressed in each statement and your personal feeling. The five points are: Strongly Disagree (1), Disagree (2), Undecided (3), Agree (4), Strongly Agree (5). You are to encircle the number which best indicates how closely you agree or disagree with the feeling expressed in each statement AS IT CONCERNS YOU.

1. I am always under a terrible strain in a math class.	1	2	3	4	5
2. I do not like mathematics, and it scares me to have to take it.	1	2	3	4	5
3. Mathematics is very interesting to me, and I enjoy math courses.	1	2	3	4	5
4. Mathematics is fascinating and fun.	1	2	3	4	5
5. Mathematics makes me feel secure, and at the same it is stimulating.	1	2	3	4	5
6. My mind goes blank, and I am unable to think clearly when working math.	1	2	3	4	5
7. I feel a sense of insecurity when attempting mathematics.	1	2	3	4	5
8. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	1	2	3	4	5
9. The feeling that I have toward mathematics is a good feeling.	1	2	3	4	5
10. Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	1	2	3	4	5
11. Mathematics is something which I enjoy a great deal.	1	2	3	4	5
12. When I hear the word math, I have a feeling of dislike.	1	2	3	4	5
13. I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.	1	2	3	4	5
14. I really like mathematics.	1	2	3	4	5
15. Mathematics is a course in school which I have always enjoyed studying.	1	2	3	4	5
16. It makes me nervous to even think about having to do a math problem.	1	2	3	4	5

17. I have never liked math, and it is my most dreaded subject.	1	2	3	4	5
18. I am happier in a math class than in any other class.	1	2	3	4	5
19. I feel at ease in mathematics, and I like it very much.	1	2	3	4	5
20. I feel a definite positive reaction to mathematics; it's enjoyable.	1	2	3	4	5

Appendix H

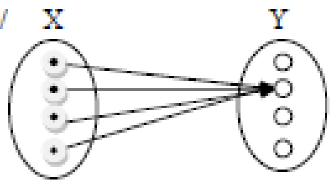
Questionnaire

Note / f (x) is a function from R ⟶ R

Task 1/ Can you give the definition of function?

Task 2/ Examine which of the following correspondences are function?

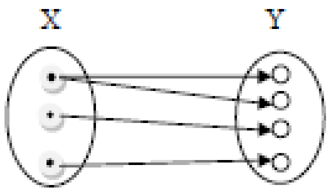
A/



Yes ☐ No ☐ I do not know ☐

Justify your answer: ------------------------------

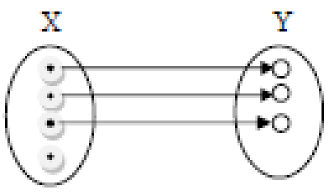
B/



Yes ☐ No ☐ I do not know ☐

Justify your answer: ------------------------------

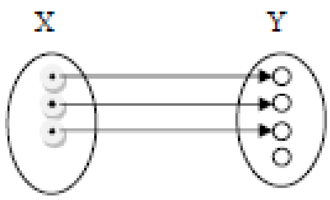
C/



Yes ☐ No ☐ I do not know ☐

Justify your answer: ------------------------------

D/



Yes ☐ No ☐ I do not know ☐

Justify your answer: ------------------------------

Task 3/ Every relation between two sets is a function. Yes ☐ No ☐ I do not know ☐

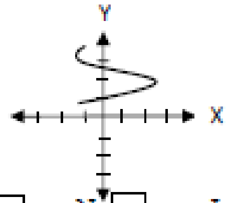
Justify your answer: ------------------------------

Task 4/ Are ($f(x) = x^2$) and ($y = x^2$) the same function? Yes ☐ No ☐ I do not know ☐

Justify your answer: ---

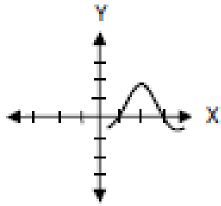
Task 5/ Do these graphs represent a function?

A/



Yes ☐ No ☐ I do not know ☐

Justify your answer: -------------------------------



Yes ☐ No ☐ I do not know ☐

Justify your answer: ---

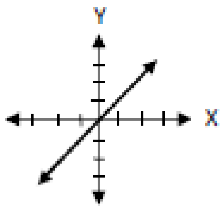
Task 6/ Do these order pairs represent a function {(2,5), (2,6) , (3,7) }?

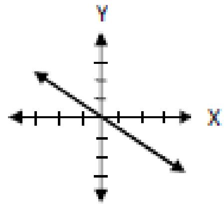
Yes ☐ No ☐ I do not know ☐

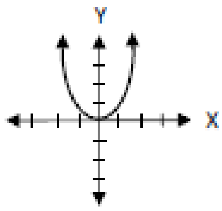
Justify your answer: ---

Task 7/ which one of these graphs represents the graph of follow functions?

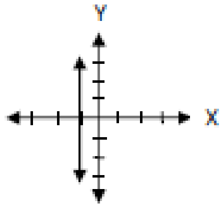
A/ $f(x) = x$

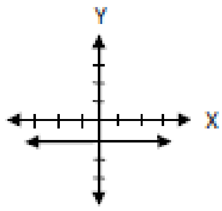


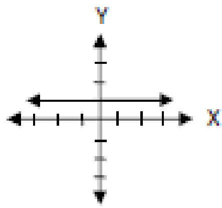




B/ $f(x) = 1$

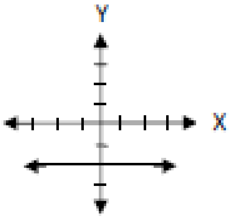






Task 8/ Find the expression of the function represented by these graphs?

A/



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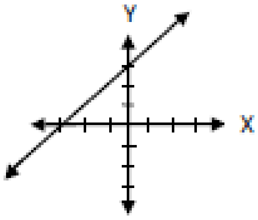
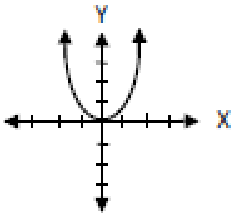


$f(x)=x-2$ ☐ $f(x)=-2$ ☐ $f(x)=x^2$ ☐

$f(x)=3$ ☐ $f(x)=x-3$ ☐ $f(x)=x+3$ ☐

Task 9/ find the graph of the function that represented by order pair {..., (1,2), (2,5), (3,10), (4,17), ...}.



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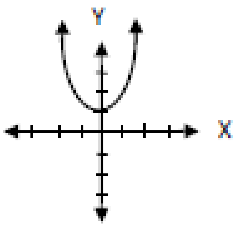

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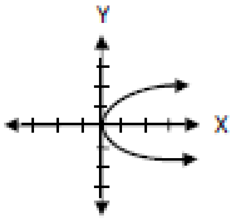

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